Solutions to EECS 206 Exam 2, 2002-11-7

Regrade requests must be submitted to Prof. Fessler or Prof. Hero by Nov. 22, with a written cover sheet explaining the request clearly. All problems will be re-examined, and scores may increase or decrease. Discussion of the exam with a professor or GSI nullifies the opportunity to submit a regrade request.
Exception: score calculation errors can be corrected during in office hours.

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1. (14)
(5pts) The DFT of "3" is 3 for k=0 and 0 otherwise. *(HW 5-2b)*
(7pts) The DFT of $(-1)^n = \mathrm{e}^{\jmath(2\pi/8)4n}$ is 1 for k=4 and 0 otherwise. *(HW 5-2d)*
(2pts for putting it all together correctly): Using linearity: *(HW 5-5c)*

$$Y[k] = \begin{cases} 4, & k=0 \\ 2, & k=4 \\ 1, & k=2,6 \\ 0, & k \text{ odd} \end{cases} = [4\ 0\ 1\ 0\ 2\ 0\ 1\ 0].$$

Few students got this problem correct, even though the parts were nearly identical to the above problems on HW5. Many students correctly or nearly correctly computed the inverse DFT of $X[k]$ to find $x[n]$. As seen from the solution above, this step was unnecessary work, and most students who took this approach got stuck and did not successfully work back to the frequency domain to find $Y[k]$. No partial credit was given for finding $x[n]$, except in the rare cases where a student correctly or very nearly correctly was able to work forward from $x[n]$ to find $Y[k]$.

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2. (6)
Not linear due to +2. (Zero input signal yields nonzero output signal.) It is causal and time-invariant. *(HW 7-5)*
(b), (c), (e). Grading: a/b, c/d, e/f scored independently.

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3. (10)
(d): The highest frequency is $f_{\max} = 15$, so any sampling frequency *greater than* 30Hz will prevent aliasing.
Grading: no partial credit. *(HW 6-1)*

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4. (10)
$f = \frac{\hat{\omega}}{2\pi} f_s = (1/4)100 = 25$, so the unaliased case is $\cos(2\pi 25t + \pi/5)$.
The two types of aliased signals are $\cos(2\pi(25 + mf_s)t + \pi/5)$ and $\cos(2\pi(-25 + mf_s)t - \pi/5)$ for any $m \in \mathbb{Z}$.
(b), (e), (f) are the only correct answers. *(HW 7-7)*
Grading: score(D) = 10 if D=0; 6 if D=1; 3 if D=2; 0 if D>=3; where D is the Hamming distance between correct solution and given solution (# of omitted and extra answers).

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5. (10)
(d): Since $1 + \jmath = \sqrt{2}\mathrm{e}^{\jmath\pi/4}$, the signal is $x(t) = 1 + 2\cos(2\pi 3t) + 2\sqrt{2}\cos(2\pi 9t + \pi/4)$. *(HW 4-1)*

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6. (10)
(f): A time shift by τ causes a phase shift of $2\pi f\tau$ for a sinusoid of frequency f. A time shift of $1/20$ causes a $2\pi 5(1/20) = \pi/2$ phase shift for the 5Hz component. *(HW 4-2d,5-5b)*

7. (10)
(b): $x(t)$ has sinusoidal components at 700, 1200, and 1700 Hz, and is periodic with $f_0 = 100$Hz. *(HW 4-3,1-4)*

8. (10)
(b): $f = (k/N)f_\text{s} = (14/32)f_\text{s}$, so $f/f_\text{s} = 7/16$. *(Lab 4)*

9. (10)
(e): $x[n] = \sum_{k=0}^{1} X[k]\, \mathrm{e}^{\jmath\,(2\pi/2)n} = X[0] + X[1]\,(-1)^n$.
So $(x[0]\,, x[1]) = (3/2 + 1/2, 3/2 - 1/2) = (2, 1)$. *(HW 5-2,5-3)*

10. (10)
(e): For $k \neq 0$, $\alpha_k = \frac{1}{4}\int_{-1}^{1} 1 \cdot \mathrm{e}^{-\jmath\,(2\pi/4)kt}\,\mathrm{d}t = \frac{1}{\jmath\,2\pi k}\left[\mathrm{e}^{-\jmath\,(2\pi/4)k} - \mathrm{e}^{\jmath\,(2\pi/4)k}\right] = \frac{1}{\pi k}\sin((\pi/2)k)$.
Thus $\alpha_5 + \alpha_{-5} = 1/(5\pi) + 1/(5\pi) = \frac{2}{5\pi}$. *(HW 4-5)*

11. (10)
Since $x[n]$ is 6-periodic, it is a sum of discrete-time sinusoids. Since there is no aliasing, each of those sinusoids originated from a continuous-time sinusoid. First we use the DFT to express $x[n]$ as a sum of sinusoids:

$$X[k] = \frac{1}{6}\sum_{k=0}^{5} x[n]\,\mathrm{e}^{-\jmath\,(2\pi/6)kn} = \frac{1}{6}[12 - 12\mathrm{e}^{-\jmath\,(2\pi/6)3k}] = 2[1 - (-1)^k].$$

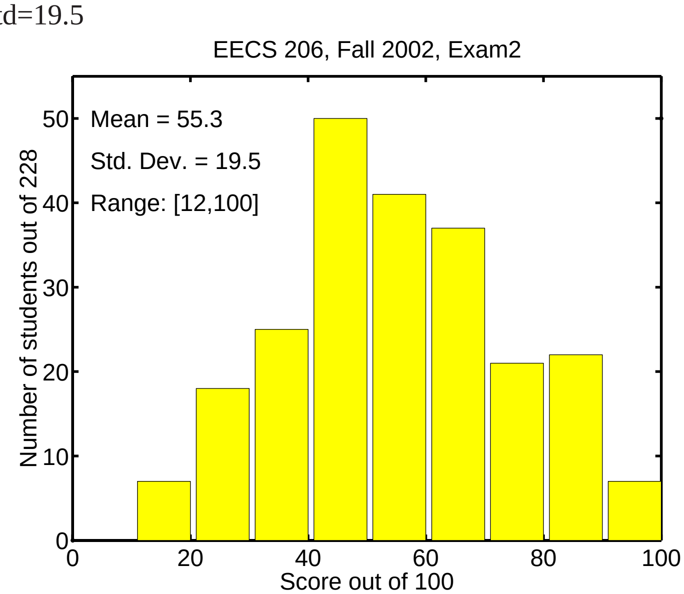
Thus using the invers DFT (synthesis equation):

$$x[n] = \sum_{k=0}^{5} X[k]\,\mathrm{e}^{\jmath\,(2\pi/6)kn} = 4\mathrm{e}^{\jmath\,(2\pi/6)n} + 4\mathrm{e}^{\jmath\,(2\pi/6)3n} + 4\mathrm{e}^{\jmath\,(2\pi/6)5n} = 8\cos((2\pi/6)n) + 4\cos(\pi n).$$

In the absence of aliasing, $\cos(2\pi ft)$ becomes $\cos(2\pi(f/f_\text{s})n)$ after sampling. Thus:

$$x(t) = 8\cos(2\pi(100/6)t) + 4\cos(2\pi(100/6)3t).$$

228 students, mean=55.3, std=19.5



For elaboration on these solutions, please come to office hours.