Name: $\qquad$
ID Number: $\qquad$
Lab section: $\qquad$
Lecture section: $\qquad$
I have neither given nor received aid on this examination, nor have I concealed any violation of the Honor Code.
Signature: $\qquad$

- This is a 120 minute exam.
- It is closed book, closed notes, closed computer.
- You may use three $8.5 \times 11$ " piece of papers, both sides, and a calculator.
- There are 12 problems. The questions are not necessarily in order of increasing difficulty.
- This exam has 6 pages. Make sure your copy is complete.
- Continuing to write anything after the ending time is announced will be considered an honor code violation. Fill out your name etc. above now.
- Clearly circle the letter(s) for your answer in this table. There are no intentional "none of the above" answers on this exam, but there is always the slim possibility of a typographical error. If you are confident that the correct answer is "none of the above" in any problem, then make a clear mark in this table and show your work clearly near that problem.
Some problems may require multiple answers (as indicated), and some partial credit may be awarded for some of these problems. For problems with only a single answer, and no partial credit will be given.

| 1. | a | b | c | d | e | f |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2. | a | b | c | d | e | f |
| 3. | a | b | c | d | e | f |
| 4. | a | b | c | d | e | f |
| 5. | a | b | c | d | e | f |
| 6. | a | b | c | d | e | f |
| 7. | a | b | c | d | e | f |
| 8. | a | b | c | d | e | f |
| 9. | a | b | c | d | e | f |
| 10. | a | b | c | d | e | f |
| 11. | a | b | c | d | e | f |
| 12. | a | b | c | d | e | f |

1. (10 points)

Determine the impulse response for the LTI system having the polezero plot shown to the right, assuming the gain $g=1$.
Check your work carefully!

a) $h[n]=\delta[n]+\delta[n-1]+\delta[n-2]+\delta[n-3]$
b) $h[n]=\delta[n]-\delta[n-1]+\delta[n-2]-\delta[n-3]$
c) $h[n]=\delta[n]+\delta[n-1]-\delta[n-2]-\delta[n-3]$
d) $h[n]=\delta[n]-\delta[n-1]-\delta[n-2]$
e) $h[n]=\delta[n]+\delta[n-1]+\delta[n-3]$
f) $h[n]=\delta[n]+\delta[n-1]-\delta[n-2]$
2. (10 points)

The 16-point DFT $X[k]$ of a signal $x[n]$ (with average power $=1.04$ ) is shown below.


To reduce data storage, an engineer is considering zeroing all DFT coefficients having magnitude less than 0.3. Let $\hat{x}[n]$ denote the signal that would be computed using the 16 -point inverse DFT of the DFT coefficients having magnitude greater than 0.3. Determine the average power of $\hat{x}[n]$.
a) 0
b) 0.16
c) 0.6
d) 0.96
e) 1
f) 1.6
3. (8 points)

A LTI system has system function $H(z)=\frac{1-z^{-1}}{1-\frac{1}{2} z^{-1}}$. The output signal is measured and has the form

$$
y[n]= \begin{cases}1, & n=0 \\ -1, & n=1 \\ 0, & \text { otherwise }\end{cases}
$$

Determine which of the following signals was the input to the filter.
a) $x[n]=\delta[n]-\delta[n-1]$
b) $x[n]=1-\frac{1}{2}(n-1)$
c) $x[n]=(-1)^{n}$
d) $x[n]=\cos (\pi n)$
e) $x[n]=(1 / 2)^{n} u[n]$
f) $x[n]=\delta[n]-\frac{1}{2} \delta[n-1]$
4. (8 points)

The input-output relation of a filter is

$$
y[n]=-\frac{1}{4} y[n-2]+x[n]+\frac{1}{4} x[n-1] .
$$

Determine which of the following is the pole-zero plot of this filter.
a)

c)

d)

e)
f)



## 5. (8 points)

An LTI system has the following system function

$$
H(z)=\frac{1+z^{-1}}{1-\frac{3}{2} z^{-1}+\frac{1}{2} z^{-2}}
$$

The impulse response $h[n]$ of such a system is the sum of one or more simple components of the form $c \delta[n]$ or $b a^{n} u[n]$. For this system, which of the following expressions are components of the impulse response $h[n]$ ?

## Circle all that apply in the answer table.

a) $\frac{1}{2} \delta[n]$
b) $4 u[n]$
c) $4(-1)^{n} u[n]$
d) $3(1 / 2)^{n} u[n]$
e) $2(1 / 4)^{n} u[n]$
f) $(-1 / 2)^{n} u[n]$
6. (8 points)

A certain application requires a filter that satisfies the design criterion $|\mathcal{H}(\pi / 2)|>|\mathcal{H}(0)|$ and the response to a sinusoid with frequency $\pi$ must be zero. Which of the following pole-zero plots corresponds to a filter that satisfies that criterion?

## Circle all that apply in the answer table.



a)

b)
c)

e)

f)
7. (8 points)

A LTI filter has the input-output relation

$$
y[n]=\frac{1}{10} y[n-1]+x[n]-\frac{9}{10} x[n-1] .
$$

The (non-ideal) magnitude response of this filter best corresponds to which of the following?
a) Coffee filter
c) Highpass filter
e) Bandpass filter
b) Notch filter
d) Lowpass filter
f) Resonator

## 8. (8 points)

Consider the following cascade of two LTI systems:

$$
x[n] \rightarrow h_{1}[n]=2 \delta[n]-\delta[n-1] \rightarrow h_{2}[n]=\delta[n]+2 \delta[n-1]+3 \delta[n-2] \rightarrow y[n] .
$$

Let $h[n]$ denote the overall impulse response of this cascade system. Determine $h[2]$.
a) 0
b) 1
c) 2
d) 3
e) 4
f) 5
9. (8 points)

An LTI filter has the pole-zero plot shown to the right with corresponding frequency response $\mathcal{H}(\hat{\omega})$. Determine $\mathcal{H}(\pi) / \mathcal{H}(0)$.
a) 0
b) 1
c) -1
d) $\sqrt{2}$
e) 2
f) $e^{j \pi / 2}$

10. (8 points)

A LTI system has impulse response $h[n]=(1 / 2)^{n} u[n]+(1 / 2)^{n} u[n-1]$.
For some unknown input signal we observe that the output of the system is $y[n]=0$.
Determine which of the following signals could have been the input to the filter.
Circle all that apply in the answer table.
a) $x[n]=0$
b) $x[n]=(-1)^{n}$
c) $x[n]=\frac{3}{2}(1 / 2)^{n} u[n]$
d) $x[n]=(1 / 2)^{n} \cos (\pi n)$
e) $x[n]=\sqrt{2} \cos (\pi n) u[n]$
f) $x[n]=\cos (\pi n+\pi / 3)$

## 11. (8 points)

The signal $x[n]=\cos \left(\frac{\pi}{2} n\right)$ is the input to an FIR filter with coefficients $b_{0}=2, b_{1}=1$, and $b_{2}=1$.
Determine which of the following could be the output signal $y[n]$.
Circle all that apply in the answer table.
a) $y[n]=\sqrt{2} \cos \left(\frac{\pi}{2} n-\frac{\pi}{4}\right)$
b) $y[n]=2 \sqrt{2} \cos \left(\frac{\pi}{2} n\right)$
c) $y[n]=0$
d) $y[n]=\cos \left(\frac{\pi}{2} n\right) u[n]$
e) $y[n]=\sqrt{2} \cos \left(\frac{\pi}{2} n+\frac{\pi}{4}\right) u[n]$
f) $y[n]=2 \cos \left(\frac{\pi}{4} n-\frac{\pi}{4}\right)$
12. (8 points)

We need a system whose response to a DC signal is zero.
Which of the following systems or combinations of systems provide this property?

- System1 has impulse response $h_{1}[n]=\delta[n]-\delta[n-1]$.
- System2 has impulse response $h_{2}[n]=\delta[n]+\delta[n-2]$.
- System3 has the pole-zero plot shown to the right:

Circle all that apply in the answer table.
a) System 1
d) System 1 cascaded with System 2
b) System 2
e) System 2 cascaded with System3
c) System3
f) System 3 cascaded with System 1


