Solutions to EECS 206 Exam 3, 2002-12-16

Regrade requests must be submitted to Prof. Fessler or Prof. Hero by Dec. 20, with a written cover sheet explaining the request clearly. All problems will be re-examined, and scores may increase or decrease. Discussing the exam with a professor or GSI nullifies the opportunity to submit a regrade request.
No partial credit was given except where indicated below. In particular, "multiple answer" problems that had only one correct answer had to be answered exactly correctly.
(There were multiple versions of the exam so the solutions below may not be in the same order as your exam.)

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1. (10)
(b): $H(z) = \dfrac{(z-j)(z+j)(z-1)}{z^3} = \dfrac{(z^2+1)(z-1)}{z^3} = 1 - z^{-1} + z^{-2} - z^{-3}$.
So $h[n] = \delta[n] - \delta[n-1] + \delta[n-2] - \delta[n-3]$. *(HW text 8.13)*

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2. (10)
(d): $\mathrm{MS}(\hat{x}) = \sum_{k=0}^{N-1} |\hat{X}[k]|^2 = (8^2+4^2+4^2)/10^2 = 0.96$ *(HW 5-4c,6-3)*

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3. (8)
(f): $X(z) = Y(z)\,/H(z) = (1-z^{-1})/H(z) = 1 - \frac{1}{2}z^{-1}$, so $x[n] = \delta[n] - \frac{1}{2}\delta[n-1]$. *(HW 9-6a)*

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4. (8)
(f): $H(z) = \dfrac{1+\frac{1}{4}z^{-1}}{1+\frac{1}{4}z^{-2}} = \dfrac{z(z+\frac{1}{4})}{z^2+\frac{1}{4}}$ which has zeros at $z = 0, -1/4$ and poles at $z = \pm j/2$. *(HW text 8.5)*

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5. (8)
(b): Answer (d) is almost correct, but has the wrong sign so it is not part of the *sum*. (This sign error was accidental.)

$$H(z) = \frac{1+z^{-1}}{1-\frac{3}{2}z^{-1}+\frac{1}{2}z^{-2}} = \frac{1+z^{-1}}{(1-z^{-1})(1-\frac{1}{2}z^{-1})} = \frac{4}{1-z^{-1}} + \frac{-3}{1-\frac{1}{2}z^{-1}},$$

so $h[n] = 4u[n] - 3(1/2)^n u[n]$. *(HW text 8.11)*
Grading: 8 for b only, 7 for b,d only, 3 for d only, 4 for b and one other (except d), 0 otherwise.

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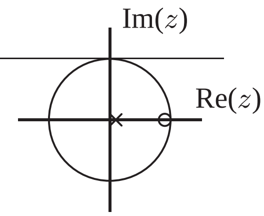
6. (8)
(b) is the only suitable choice. *(HW text 8.16)*
Grading: 8 for b only or b and e, 0 otherwise.
The intention was for this problem to solvable by "eyeballing" the distances rather than by exact calculation. However, answer (e) was too subtle to eyeball. Assuming the pole is at $z = -1/2$, one can show that $|\mathcal{H}(\pi/2)\,| = \sqrt{16/10}$ whereas $|\mathcal{H}(0)\,| = \sqrt{16/9}$, so (e) is incorrect. Because this difference is so subtle , we ignored answer (e) in grading. For completeness, we note that more generally, if the pole is at $z = -a$, then $|\mathcal{H}(\pi/2)\,| = \sqrt{\frac{2}{1+a^2}}$ and (for $a \neq 1$):

$$|\mathcal{H}(0)\,| = \sqrt{\frac{4}{(1+a)^2}} = \sqrt{\frac{2}{1+a^2} + \frac{2(1-a)^2}{(1+a)^2(1+a^2)}} > |\mathcal{H}(\pi/2)\,|.$$

So answer (e) does not satisfy the design criterion no matter where the pole is located. Nevertheless, we chose not to penalize students who eyeballed it incorrectly and included (e) in their answer.



7. (8)
(c): The system function is $H(z) = (1 - \frac{1}{10}z^{-1})/(1 - \frac{9}{10}z^{-1})$ and the pole-zero plot is shown to the right. This is a highpass filter since the magnitude response at high frequencies is much higher than the response at low frequencies. *(HW text 8.20d)*

8. (8)
(e). By convolution (or multiply z-transforms) we get $h[n] = \cdots + 4\delta[n-2] + \cdots$ so $h[2] = 4$.. *(HW 8-8d)*
Grading: 8 if correct, 5 if 1 error, 2 if 2 errors, 0 otherwise. (An error is an omission or an extra circle.)

9. (8)
(c): $H(z) = g\dfrac{(z-j)(z+j)}{z^3} = g\dfrac{(z^2+1)}{z^3} = g(z^{-1} + z^{-3})$.
So $\mathcal{H}(\hat{\omega}) = g(\mathrm{e}^{-\jmath\hat{\omega}} + \mathrm{e}^{-\jmath 3\hat{\omega}})$, $\mathcal{H}(0) = 2$, $\mathcal{H}(\pi) = -2$, and $\mathcal{H}(\pi)\,/\mathcal{H}(0) = -1$. *(HW text 8.20)*

10. (8)
(a): $h[n] = (1/2)^n u[n] + \frac{1}{2}(1/2)^{n-1}u[n-1]$ so $H(z) = \dfrac{1}{1-\frac{1}{2}z^{-1}} + \dfrac{1}{2}z^{-1}\dfrac{1}{1-\frac{1}{2}z^{-1}} = \dfrac{1+\frac{1}{2}z^{-1}}{1-\frac{1}{2}z^{-1}}$.
$H(z)$ is zero at $z = -1/2$, which is not on the unit circle, so no sinusoids are nulled. *(HW 9-6)*

11. (8)
(a): $\mathcal{H}(\hat{\omega}) = 2 + \mathrm{e}^{-\jmath\hat{\omega}} + \mathrm{e}^{-\jmath 2\hat{\omega}}$ so $\mathcal{H}(\pi/2) = 2 - \jmath - 1 = \sqrt{2}\mathrm{e}^{-\jmath\pi/4}$ so $y[n] = \sqrt{2}\cos(\frac{\pi}{2}n - \frac{\pi}{4})$. *(HW text 6.4)*

12. (8)
(a),(d),(f) since $\mathcal{H}_1(\hat{\omega}) = 1 - \mathrm{e}^{-\jmath\hat{\omega}}$ so $\mathcal{H}_1(0) = 0$. But $\mathcal{H}_2(0) \neq 0$ and $\mathcal{H}_3(0) \neq 0$. *(HW 8-8 (cascade), text 7.9)*

223 students, mean=73.4, std=19.9