Homework #4, EECS 206, Fall 2002. Due Fri. Oct. 11, by 4:30PM

Notes

- Review the HW policies on HW1!
- Reading: "Part 3" lecture notes, Ch. 3 of text, 3.4.5 supplement (on web), Prof. Wakefield's Fourier series "quick primer" (on web)

Skills and Concepts

- spectra of sums of sinusoids
- spectra of periodic signals (Fourier series)

Problems

1. [20] Text 3.3. (plot to signal, points: a10, b5, c5)

2. [30] Consider the signal $x(t) = 4 + \cos(2\pi 3t) + \sin^3(5\pi t)$.
   (a) [10] Express this $x(t)$ as a sum of complex exponential signals. (Use an inverse Euler identity.)
   (b) [10] Plot the spectrum (magnitude and phase) of this signal.
   (c) [10] Now consider the signal $y(t) = x(2t)$, and plot the magnitude spectrum of $y(t)$.
   (d) [0] Describe how this time scaling affected the spectrum. What about a time shift?

3. [20] Text 3.6. (AM radio)

4. [30] One of the important topics that we will discuss later in the course is **filtering**, which means processing a signal in a way that amplifies or attenuates various frequency components. The tone controls (bass and treble) in a stereo system are examples of filters: when you "turn up the bass" you are amplifying the low frequency components, and when you "turn down the treble" you are attenuating the high frequency components. This problem is a preview of filters.

   You will analyze what happens when a signal $x(t)$ passes through a special kind of filter called a **moving average** filter. This filter is used frequently in signal analysis to "smooth out" signals. A block diagram for a filter looks like

   $$\text{(Input) } x(t) \to \boxed{\text{filter}} \to y(t) \text{ (Output).}$$

   For this problem, assume that the input signal $x(t)$ has the spectrum described by the following (amplitude, frequency) pairs $\left\{(2+\jmath 2, -8), (4e^{-\jmath \pi/3}, -3), (1, 0), (4e^{\jmath \pi/3}, 3), (2-\jmath 2, 8)\right\}$, where the frequencies are in Hz.

   (a) [5] Plot the magnitude spectrum of $x(t)$.

   (b) [10] Express $x(t)$ as a sum of sinusoidal signals. Hint: there are three terms, and one of the amplitudes is $4\sqrt{2}$.

   (c) [10] A T-second moving average filter is described by the following formula:

   $$y(t) = \frac{1}{T} \int_0^T x(t+\tau) \, d\tau.$$

   Using this formula with $T = 1/4$ s and the formula for $x(t)$ derived in the previous part, determine $y(t)$. Your integration should give you a sum of a few sinusoidal signals and you should simplify the result using phasor methods.

   (d) [5] Plot the magnitude spectrum of $y(t)$.

   (e) [0] Describe qualitatively how the spectrum of the output signal compares to the spectrum of the input signal.

   (f) [0] Repeat the above for a generic sinusoidal signal $\cos(2\pi f t)$, and make a plot of the output amplitude as a function of the input frequency f. Does this filter amplify or attenuate the high or low frequency components?

5. [20] Problem 2 in Part 3 lecture notes (Fourier series of simple periodic signals)

   Part (a) will not be graded. The answer to part (a) is $\alpha_k = T_0 \cdot \begin{cases} 1/2, & k = 0, \\ \frac{\jmath}{2\pi k}, & k \neq 0. \end{cases}$

   To do the integral using matlab's symbolic toolbox, you could try the following command.

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pretty(int('t * exp(-i*2*pi*k*t)', 't', 0, 1))
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6. [25] Problem 4 in Part 3 lecture notes (signal properties from spectra)

7. [5] Problem 6 in Part 3 lecture notes (power of sum of harmonic sinusoids)

   Hint: the very hard way to do this would be to use the original power formula in the Part 1 lecture notes. Don't do it that way!

8. [0] Problem 10 in Part 3 lecture notes (Fourier series of even and odd signals)