Homework #5, EECS 206, Fall 2002. Due **Fri. Oct. 18**, by 4:30PM

Notes

- Review the HW policies on HW1!
- Reading: "Part 4" lecture notes. (This is the primary source.)
  As needed: Text 4.1.1, 9.1.1, 9.3.1, 9.3.2, Section 3 in Prof. Wakefield's "Primer on DFT."

Skills and Concepts

- discrete-time sinusoidal signals
- spectra of sums of discrete-time sinusoids
- spectra of discrete-time periodic signals (discrete Fourier series)
- DFT

Problems

0. [0] The alternate exam time for Exam 2 on Nov. 7 is 4-5:30. Visit the web page under "Exams" to sign up for the alternate time if you have a conflict with the regular exam time, which is 6-7:30PM. Email Prof. Hero *only if* you have a conflict with *both* the regular and alternate exam time. You must sign up for the alternate exam time **before Oct. 31.**

1. [10] (a) [0] Show that the 16-point DFT of the signal $x[n] = 1 + 4\cos(3\pi n/4 + \pi/5)$ is given by

$$X[k] = \begin{cases} 1, & k = 0 \\ 2e^{\jmath\pi/5}, & k = 6 \\ 2e^{-\jmath\pi/5}, & k = 16 - 6 = 10 \\ 0, & \text{otherwise}. \end{cases}$$

(b) [5] Determine the 32-point DFT of the following signal:

$$y[n] = 29 + 8\cos(2\pi n/32) + 3\sin(2\pi 5n/32) + 8\sin(2\pi 33n/32).$$

Hint: think carefully about that "33."

(c) [5] Sketch the one-sided spectrum of the signal $y[n]$.

(d) [0] If the 32 (nonzero) values of $y[n]$ were stored in computer memory using 4-byte floating point numbers, how many bytes would be required?
Alternatively, if the list of (frequency, complex amplitude) pairs (those with nonzero amplitude) were instead stored, how many bytes would be required? (Remember that complex numbers require two values to be stored. However, there is complex conjugate symmetry in the spectrum so you could be plan to be efficient in your storage scheme.
You should find that for this type of signal the frequency domain representation is much more efficient for storage; these memory savings are fundamental to how MP3 digital audio compression works.

2. [20] Determine the N-point DFTs of the following signals.

(a) [5] $s[n] = \begin{cases} 1, & n = 0 \\ 0, & n = 1, 2, \ldots, N-1 \end{cases}$

(b) [5] $x[n] = 1$ for $n = 0, 1, \ldots, N-1$

(c) [5] $y[n] = \sin(2\pi mn/N)$ for $n = 0, 1, \ldots, N-1$, where m is one of the values in the set $\{0, 1, 2, \ldots, N-1\}$. Hint: use Euler!

(d) [5] $z[n] = \begin{cases} 1, & n \text{ even} \\ 0, & n \text{ odd}. \end{cases}$ (Assume N is even here.) Hint: $z[n] = \frac{1+(-1)^n}{2}$ and $-1 = e^{\jmath\pi}$.

3. [10] The N-point DFT of a N-periodic signal $x[n]$ is $X[k] = \begin{cases} 1, & k = 0 \\ 2, & k = 2, N-2 \\ 0, & \text{otherwise.} \end{cases}$

   Determine $x[n]$.

4. [15] The DFT coefficients of a N-periodic real signal $x[n]$ are all zero except for $X[3]$ and $X[N-3]$. For this particular signal, we know that $X[3] = A e^{\jmath\phi}$.
   (a) [5] Determine $M(x)$.
   (b) [5] Determine $x[n]$. Hint: first find $X[N-3]$.
   (c) [5] Determine the average power of $x[n]$.

5. [10] Show the following DFT properties using the synthesis equation for the DFT:

$$x[n] = \sum_{k=0}^{N-1} X[k] \, e^{\jmath (2\pi/N) kn}.$$

   (a) [0] $x[n+N] = x[n]$
   (b) [0] If $y[n] = x[n-n_1]$ then $Y[k] = X[k] \, e^{-\jmath (2\pi/N) k n_1}$.
   (c) [10] Suppose a N-periodic signal $x[n]$ has DFT $X[k] = 2^k$, $k = 0, \ldots, N-1$.
   Determine the DFT of the signal $y[n] = 7 + 3x[n-5]$.

6. [20] (Optional extra credit challenge problem. No help will be given in office hours for such problems.)
   (a) [10] Suppose $x(t)$ is T-periodic and has Fourier series coefficients α_k. Define $y(t) = \cos(2\pi t/T) x(t)$, and let β_k denote its Fourier series coefficients. Determine how to express each β_k in terms the α_k's.
   (b) [10] Apply your result to find the Fourier series of $y(t) = |\cos(2\pi t/T)| = \cos(2\pi t/T) x(t)$ where $x(t)$ is an appropriate square wave that alternates between +1 and -1.