Homework #7, EECS 206, Fall 2002. Due **Fri. Nov. 1**, by 4:30PM

Notes

- Review the HW policies on HW1!
- Last reminder for the **alternate exam signup**: see HW5.
- **Exam 2** is on November 7.
  It will cover everything up through Lab 5 and HW7, meaning all lecture notes and chapters 1-5.
  For Exam 2, you can have two 8.5x11" "**cheat sheets**," both sides. Bring your **calculator**.
- This HW will not be graded before Exam 2, so you should **copy** it for studying and for comparing to the solutions.
- Reminder: **Quiz 3** will be Oct. 29-31. It covers Labs 3 and 4.
- Reading: Text sections 5.1-5.9
- Relevant practice problems on the DSP CDROM: 5.3, 5.12-17, 5.18-29

Skills and Concepts

- FIR / LTI filters, impulse response

Several of the textbook problems have initial parts, typically part (a), that ask you to plot certain signals. You should do those plots to help you visualize the signals before proceeding to solve the rest of the problem, even though in most cases those plots will not be graded.

Problems

1. [15] Text 5.1bcd. (running average for unit step)
2. [15] Text 5.2. (output of difference equation LTI system)
   For part (c), give both a formula and a plot for $h[n]$.
3. [10] Text 5.5cd. (geometric series through running average)
4. [10] Text 5.6. (FIR coefficients from $h[n]$)
5. [20] Text 5.7. (system properties from input-output relationship)
6. [10] Text 5.8b. (using LTI properties)
7. [0] Text problem 5.11 is postponed until HW 8.
   Do not turn it in with HW7 since you will need it to turn it in with HW8!

(see next page)

8. [25] You have seen in class that the impulse response of a linear time invariant (LTI) filter completely describes its input-output properties. The impulse response is very useful since it can be used to find the output of the LTI system due to *any* arbitrary discrete-time (DT) input signal. This is possible since any DT signal can be represented as a "superposition" of weighted impulse functions:

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\, \delta[n-k].$$

Sometimes the **step response** is used to describe a LTI filter instead of the **impulse response**. For a LTI filter described by the usual input-output relation

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k] = h[n] * x[n]\,,$$

we define the step response as

$$y_{\text{step}}[n] = h[n] * u[n]\,,$$

where $u[n]$ denotes the unit step function defined in textbook Problem 5.5.

(a) [5] Show that the step response is related to the impulse response by

$$h[n] = y_{\text{step}}[n] - y_{\text{step}}[n-1].$$

Thus the step response provides just as good of a description of the LTI filter as the impulse response. Hint: express $\delta[n-k]$ as a difference between two steps.

(b) [5] Using the previous hint, show that any discrete time signal $x[n]$ can be represented as a superposition of weighted unit step functions:

$$x[n] = \sum_{k=-\infty}^{\infty} a_k u[n-k]$$

for suitable a_k.

(c) [5] Find the impulse and step responses of the FIR filter given by the input-output relation

$$y[n] = 2x[n] - 3x[n-1] + 2x[n-2]\,,$$

and verify the relation you showed in part (a).

(d) [5] Plot and compare the impulse response to the step response.

(e) [5] Plot the difference $u[n] - u[n-6]$ to see that the finite width pulse input signal

$$x[n] = \begin{cases} 1, & n = 0, 1, \ldots, 5 \\ 0, & \text{otherwise} \end{cases}$$

can be represented as $x[n] = u[n] - u[n-6]$.
Now find the output $y[n]$ of the FIR filter described in part (c) when the input is the above pulse signal $x[n]$. To do this, use the superposition property: $y[n] = y_{\text{step}}[n] - y_{\text{step}}[n-6]$, and graph the output signal $y[n]$ using the graphs of the step response $y_{\text{step}}[n]$, found in part (c), and its delayed version.
Note: for this input signal, using the step response and the superposition property provides a much quicker computation of the output signal than using the impulse response would.