Digitization and Reconstruction of Continuous Time Signals

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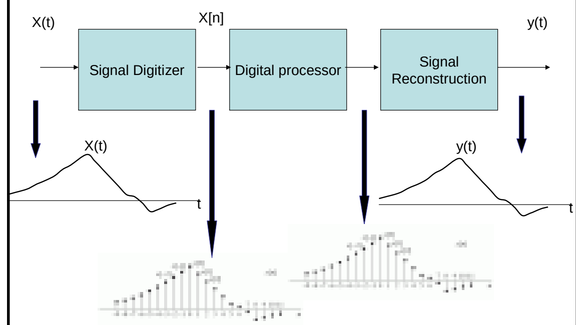
EECS206 F02
Lect 19

- Digitization/Reconstruction block diagram
- Reconstruction methods
- Reconstruction via interpolation
- Examples

Digital Signal Processing System

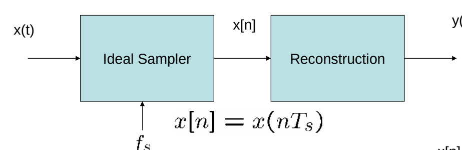


Sources of ambiguity and error

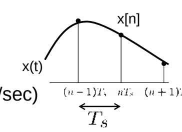
- Signal Digitization:
  – Sampling rate too slow
  – Quantization too coarse
- Digital processing:
  – Roundoff errors/register overflow
  – Transmission/memory-storage errors
  – Buggy code!
- Signal Reconstruction:
  – Interpolation errors

Sampling Considerations

- Consider time sampling/reconstruction without quantization:



$x[n] = x(nT_s)$



- T_s sampling period (secs/sample)
- f_s sampling rate or frequency (samples/sec)

$$f_s = 1/T_s$$

Reconstruction of periodic signals from their time samples

- Time domain methods: estimate x(t) by simple ZOH or by applying interpolation to x[n]

$$y(t) = \sum_{n=-\infty}^{\infty} x(nT_s)p(t - nT_s)$$

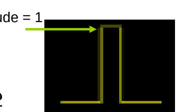
- Frequency domain methods: estimate line spectrum of x(t) from line spectrum of x[n] and use estimate to re-synthesize x(t).

Interpolation Methods of Reconstruction

- Square interpolation

$p(t) = 1, \quad -T_s/2 \le t \le T_s/2$

Amplitude = 1

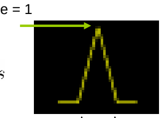


$-T_s/2 \quad T_s/2$

- Triangular interpolation

$p(t) = 1 - |t|/T_s, \quad -T_s \le t \le T_s$

Amplitude = 1

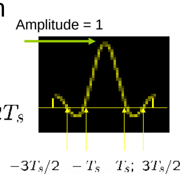


$-T_s \quad T_s$

Interpolation Methods of Reconstruction

- Truncated sinc interpolation

$$p(t) = \text{sinc}(\pi t/T_s), \quad -2T_s \le t \le 2T_s$$



$$\text{sinc}(u) = \frac{\sin(u)}{u}, \quad -\infty < u < \infty$$

Example

- Consider cts time periodic signal

$$\begin{aligned} x(t) &= 6\cos(600\pi t) \\ &\quad +6\cos(1200\pi t - \pi/3) \\ &\quad +4\cos(1800\pi t + \pi/4) \end{aligned}$$

- Fundamental frequency: ?
- Highest freq component: ?

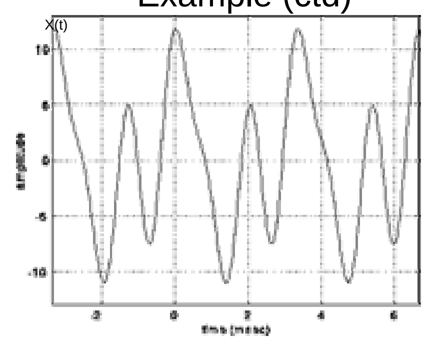
Example

- Consider cts time periodic signal

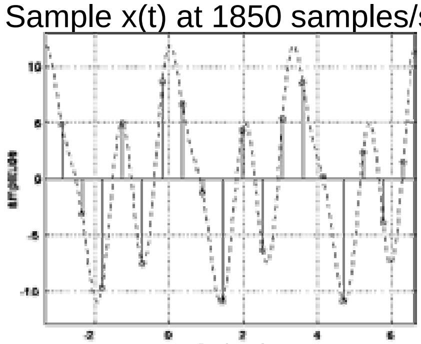
$$\begin{aligned} x(t) &= 6\cos(600\pi t) \\ &\quad +6\cos(1200\pi t - \pi/3) \\ &\quad +4\cos(1800\pi t + \pi/4) \end{aligned}$$

- Fundamental frequency: $f_o = 300$ Hz
- Highest freq component: $f_{max} = 900$ Hz

Example (ctd)

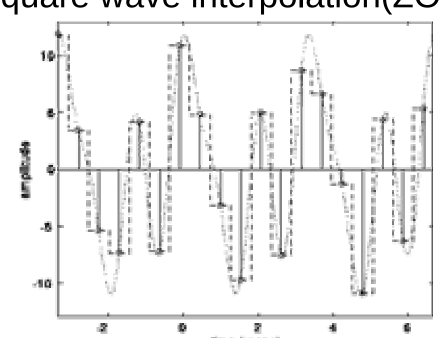


Sample x(t) at 1850 samples/sec



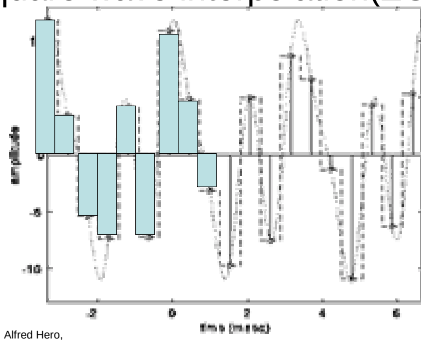
Note: max frequency sinusoid gets sampled at 1850/900=2.05 samples/cycle

Square wave interpolation(ZOH)

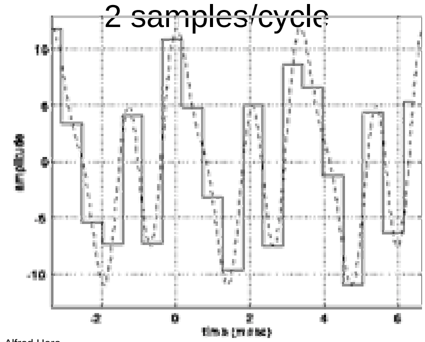


(Note sampling is time shifted wrt last slide)

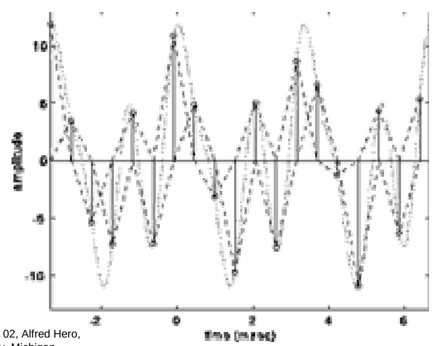
Square wave interpolation(ZOH)



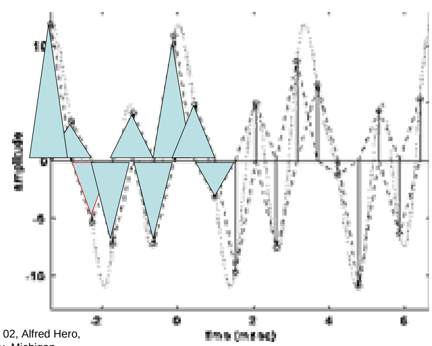
ZOH Reconstruction: 2 samples/cycle



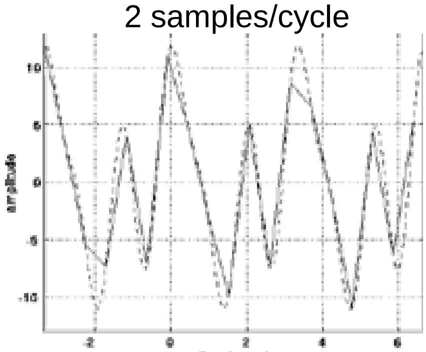
Triangular wave (linear) interpolation



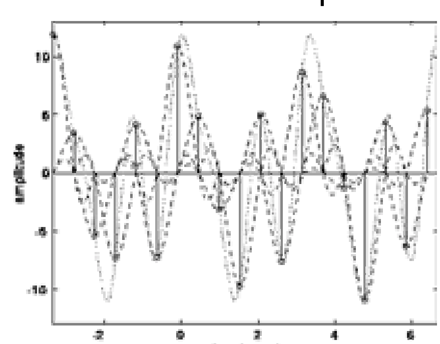
Triangular wave (linear) interpolation



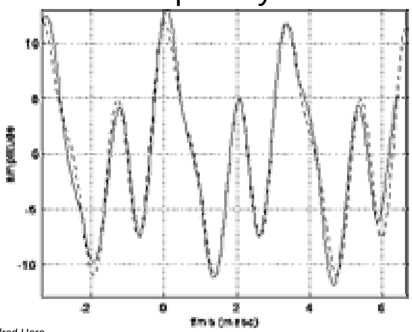
Linear interpolator reconstruction: 2 samples/cycle



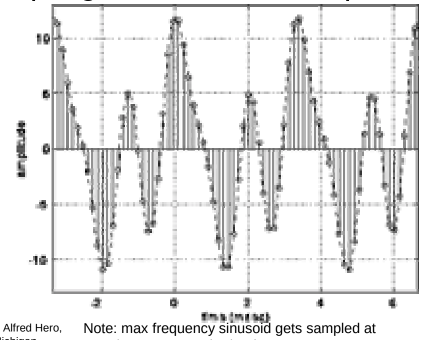
Truncated Sinc Interpolation



Truncated sinc reconstruction 2 samples/cycle

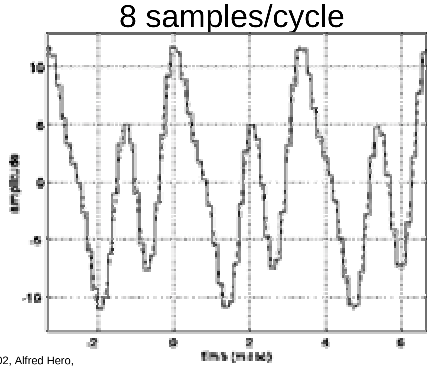


Can reduce error by increasing sampling rate to 7250 samples/sec

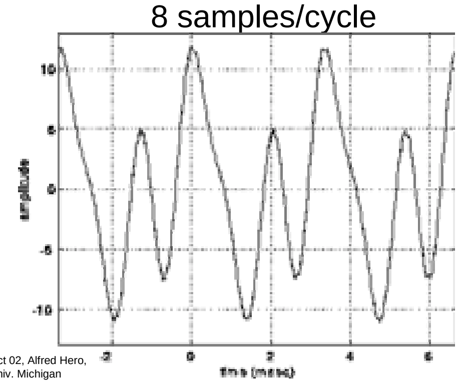


Note: max frequency sinusoid gets sampled at 7250/900=8.0 samples/cycle

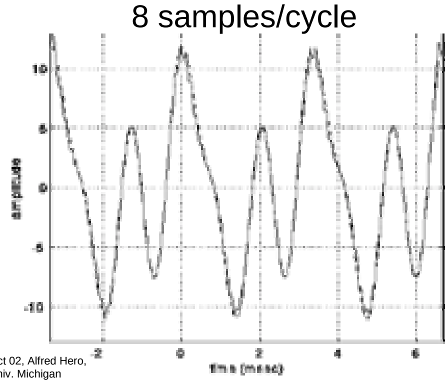
ZOH Reconstruction 8 samples/cycle



Linear interpolator reconstruction 8 samples/cycle



Truncated Sinc Reconstruction 8 samples/cycle



Poor choice of sample frequency and/or interpolation function cause errors

- Sampling a signal below a certain rate (the Nyquist rate) depending on the signal dynamics leads to fundamental ambiguity due to a phenomenon called “aliasing.”
- Over-sampling a signal above Nyquist rate never leads to aliasing but may be computationally expensive.
- Even if adequately sampled, use of poor interpolators can lead to distortion in reconstructed signal.
- There is a distortion tradeoff between low sampling rate and low complexity of interpolator