Effects of Signal Operations on Signal Characteristics

So far we have covered the following
- Signal characteristics (duration, energy, etc., about 10)
- Signal operations (amplitude scaling, etc., about 10)

It is logical to expect that performing **operations** (even simple ones!) on signals will change their **characteristics**.
So we could now derive about $10 \times 10 = 100$ "properties" that describe the effect of operation X on characteristic Y.
Instead, we will work a couple examples that illustrate the *methods* one can use to derive such properties when needed.

Example. Operation: time scaling. Property: duration.

Suppose $x(t)$ is a finite-duration signal with support interval $[t_1, t_2]$.
Let $y(t) = x(2t)$. Find the duration of $y(t)$.

Answer.
Since $x(t)$ has support interval $[t_1, t_2]$, its nonzero values occur when $t_1 \leq t \leq t_2$.
(Using what we are given about "input signal.")
Since $y(t) = x(2t)$, its nonzero values occur when $t_1 \leq 2t \leq t_2$.
(Using what we know about relationship between y and x.)
Rearranging, we see that the nonzero values of $y(t)$ occur when $t_1/2 \leq t \leq t_2/2$.
(Using math.)
Thus, the **duration** of $y(t)$ is $t_2/2 - t_1/2 = \frac{1}{2}(t_2 - t_1)$, so duration(y) = duration(x) / 2.

Example. Operation: time scaling. Property: energy.

If $y(t) = x(-2t)$, relate the energy of $y(t)$ to the energy of $x(t)$.
For simplicity we consider a finite-support signal $x(t)$, with support $[t_1, t_2]$.
By similar argument as above, the support interval of $y(t)$ is $[-t_2/2, -t_1/2]$. So the energy $E(y)$ is given by:

$$\begin{aligned}
E(y) &= \int_{-t_2/2}^{-t_1/2} y^2(t)\,\mathrm{d}t && \text{(Definition)} \\
&= \int_{-t_2/2}^{-t_1/2} x^2(-2t)\,\mathrm{d}t && \text{(Substitute given relationship)} \\
&= \int_{t_2}^{t_1} x^2(t')\,\frac{\mathrm{d}t'}{-2} && \text{(Calculus: let } t' = -2t\text{)} \\
&= \int_{t_1}^{t_2} x^2(t')\,\frac{\mathrm{d}t'}{2} && \text{(Calculus: exchanging limits)} \\
&= \frac{1}{2}\int_{t_1}^{t_2} x^2(t')\,\mathrm{d}t' = \frac{1}{2}E(x) && \text{(Using energy definition again)}
\end{aligned}$$

Why the calculus line? Because we want to make the integral look like the formula for energy of $x(t)$.

Exercise: show the following. If $y(t) = x(ct)$ for $c \neq 0$, then $E(y) = \frac{1}{|c|}E(x)$.

Can one memorize all 100 such properties? Can one "cram" them in before an exam? Unlikely.
Instead, one must learn these methods by working problems, paying attention to the tools used to find the solutions, so as to be able to apply those tools when needed for future problems.

Example. Operation: signal addition. Property: energy. (Relate $E(x+y)$ to $E(x)$ and $E(y)$ and ?.)

$$E(x+y) = \sum_n (x[n] + y[n])^2 = \sum_n x^2[n] + 2\sum_n x[n]y[n] + \sum_n y^2[n] = E(x) + 2\sum_n x[n]y[n] + E(y).$$

Correlation!