1. **C.** $Y(z)[1+2z^{-1}+3z^{-2}] = X(z)[4+5z^{-1}+6z^{-2}] \to H(z) = \frac{Y(z)}{X(z)} = \frac{4z^2+5z+6}{z^2+2z+3}$.

2. **E.** $H(z) = \frac{(z-0)(z-3)}{(z-1)(z-2)} = \frac{z^2-3z}{z^2-3z+2}$ to a constant.

3. **E.** $H(z) = \mathcal{Z}\{h[n]\} = \frac{z}{z-1} + \frac{z}{z-2} = \frac{2z^2-3z}{z^2-3z+2}$.

4. **C.** $H(z) = \mathcal{Z}\{y[n]\}/\mathcal{Z}\{x[n]\} = [\frac{2z^2-3z}{z^2-3z+2}]/[\frac{z}{z-2}] = \frac{2z-3}{z-1}$.

5. **C.** $H(e^{j\omega}) = 1+2e^{-j\omega}+3e^{-j2\omega} \to H(z) = 1+2z^{-1}+3z^{-2} = \frac{z^2+2z+3}{z^2}$.

6. **D.** $1+2z^{-1}+3z^{-2}+\frac{z}{z-1} = \frac{z^2+2z+3}{z^2}+\frac{z}{z-1} = \frac{2z^3+z^2+z-3}{z^3-z^2}$.

7. **E.** $H(e^{j\omega}) = 1+e^{-j\omega}+e^{-j2\omega}+e^{-j3\omega}$. Plugging in $\omega = \pi/2, \pi$:
$H(e^{j\pi/2}) = 1-j-1+j = 0$. $H(e^{j\pi}) = 1-1+1-1 = 0$. Output=0!

8. **C.** Notch filter with $h[1] = -1 = -2\cos\omega_0 \to \omega = \pm\pi/3$.

9. **C.** Notch filter with $h[1] = -2\cos(2\pi\frac{60}{240}) = 0$.

10. **B.** $\cos(2\pi\frac{m}{N})$ and $\cos(2\pi\frac{n}{N})$ have correlation=0 if $m \neq n$.
$\cos(2\pi\frac{m}{N})$ and $\sin(2\pi\frac{n}{N})$ have correlation=0 *even if* $m = n$.
Third time's the charm for this problem? Periodic exam problem?

11. **B.** $\sin(3n)$ just multiplies $x[n]$. Don't confuse with $\sin(3x[n])$.
But $y[n-1] = \sin(3n-3)x[n-1] \neq \sin(3n)x[n-1]$, so NOT TI.

12. **C.** $y[n]$ depends on $x[n+1]$ so not causal. FIR, so is stable. S NOT C.

13. **B.** $H(z) = \frac{3+4z^{-1}}{1+2z^{-1}} = \frac{3z+4}{z+2}$ $\to$pole at -2$\to$C NOT S.

14. **C.** $z_o = 5e^{j0.93}[\sqrt{2}e^{j0.78}]^n \to z_o + z_o^* = 2Re[z_o] = 10(\sqrt{2})^n\cos(0.78n+0.93)$.

15. **A.** Zero at $\{0.95\}$ $\to$gain low at $\omega = 0$. Pole at $\{-0.95\}$ $\to$gain high at $\omega = \pi$.

16. $H(z) = \frac{z-0.25}{(z-0.5)(z+0.25)} = \frac{1/3}{z-0.5} + \frac{2/3}{z+0.25} \to h[n] = (\frac{1}{3}(0.5)^{n-1} + \frac{2}{3}(-0.25)^{n-1})u[n-1]$.
C. But note (c) was incorrectly advanced in time by 1! Sorry about that. **Better:**

16. $\frac{1}{z}H(z) = \frac{z-0.25}{z(z-0.5)(z+0.25)} = \frac{2}{z} + \frac{2/3}{z-0.5} - \frac{8/3}{z+0.25} \to H(z) = 2 + \frac{2}{3}\frac{z}{z-0.5} - \frac{8}{3}\frac{z}{z+0.25}$
$\to h[n] = 2\delta[n] + (\frac{2}{3}(0.5)^n - \frac{8}{3}(-0.25)^n)u[n]$ agrees with the above answer (try it).

17. **A.** $H(z) = \frac{1}{4}(\frac{z}{z-0.9} + \frac{z}{z+0.4}) = \frac{1}{4}\frac{2z^2-0.5z}{(z-0.9)(z+0.4)} \to 2z^2 - 0.5z = 0 \to$ zeros at $\{0, 0.25\}$.

18. **D.** Input component at $\omega = \pm 0.1\pi$ cancelled$\to$zeros at $\{e^{\pm j0.1\pi}\}$

19. **C.** $H_1(z)H_2(z) = \frac{(z-0.9e^{j0.5\pi})(z-0.9e^{-j0.5\pi})}{(z-0)(z-0.2)}\frac{z}{z+0.9} \to$ zeros at $\{0.9e^{\pm j0.5\pi}\}$ (NOT (a))
and poles at $\{0.2, -0.9\}$ (NOT (b): poles at $\{0.9, -0.2\}$).

20. **C.** $H(z) = \frac{cz}{z-p} \to h[n] = cp^n u[n]$. $10 = cp^0$. $0.04343 = 10p^{10} \to p = 0.580$.

21. **D.** $Y(z)(1+\sum_{k=1}^{9} z^{-k}) = X(z) \rightarrow H(z) = \frac{Y(z)}{X(z)} = z^9/[\sum_{k=0}^{9} z^k]$.
System NOT low-pass since no pole at $z=1 \Leftrightarrow \omega = 0$. Of course, it's also unstable!

22. **B.** $H(z) = \frac{UNKNOWN}{1-az^{-1}+z^{-2}} \rightarrow H(z)$ has poles at zeros of $1+az^{-1}+z^{-2}$.
Recall (?) notch filter with zeros at $e^{\pm j\omega_o}$ has $G(z) = 1 - 2\cos(\omega_o)z^{-1} + z^{-2}$.
$|a| < 1$ ensures $|-2\cos(\omega_o)| < 2$ so $H(z)$ is an inverse notch filter (resonator).

23. **D.** $H(z) = \frac{1}{1-0.9e^{j\theta_o}z^{-10}} = \frac{z^{10}}{z^{10}-0.9e^{j\theta_o}} \rightarrow$ poles at $\{0.9^{0.1}e^{j0.1(\theta_o+2\pi k)}, k = 0\ldots 9\}$.

24. **B or C.** Zeros at $\{e^{\pm j0.1\pi}, e^{\pm j0.11\pi}\} \rightarrow$ reject roughly $0.1\pi < \omega 0.11\pi$
This isn't really *band*-reject; more like *notch*-reject, but we'll accept (b).
$H(z) = (z-e^{j0.1\pi})(z-e^{-j0.1\pi})(z-e^{j0.11\pi})(z-e^{-j0.11\pi})/z^4 \rightarrow 4$ poles at $z=0$.

25. **A.** Only (a) is true. (c) is almost true: $y[n]$ has $4+4-1=7$ nonzero values.

26. **B.** $H(z) = \frac{Y(z)}{X(z)} = [\frac{5z^2-0.1z+1}{z^2-z}]/[\frac{z}{z-1}] = \frac{5z^2-0.1z+1}{z^2}$. Note order of (c) is reversed.

27. The rectified sine $|H(e^{j\omega})| = |\sin(2\omega)|$ was worth 8/10 (many got this score).
This is the correct answer only if the poles are all neglected, so it's pretty generous.
To get 10/10, I wanted sharp peaks at $\omega = \pm\pi/4$ and $\omega = \pm 3\pi/4$. See below.

28. $H_3(z) = H_1(z)H_2(z) = [\frac{z}{z-0.4}][(1-0.4z^{-1}] = 1 \rightarrow h_3[n] = \delta[n]$. **OR:**
$h_3[n] = h_1[n] * h_2[n] = h_1[n] - 0.4h_1[n-1] = (0.4)^n u[n] - (0.4)(0.4)^{n-1}u[n-1] = \delta[n]$.

EXAM SCORES BY LECTURE SECTION–SEE WHERE YOU STAND

#1: $143, 140^6, 139, 138, 135^3, 133^4, 130^2, 128, 125, 123^2, 120^6, 118^2, 113^3, 112, 110, 109, 108, 105, 104^2, 103^2, 100, 98^5, 95^2, 93, 92, 91^2, 88^2, 87, 86, 83^2, 80^2, 78, 77, 72, 70, 65, 55, 45, 22$
Median: 110. Mean: 107.4. #: 70.

#2: $144, 140^2, 139, 138^5, 137, 135^3, 134^3, 133^3, 130^4, 129^2, 128^3, 125^4, 124^4, 123^4, 120^4, 119^3, 118^2, 115^3, 114, 113^3, 112, 110^3, 108^2, 105^2, 104^2, 103, 102, 100^4, 98^7, 93^2, 91, 80, 73, 72, 68, 64, 63, 46$.
Median: 120. Mean: 114.9. #: 89. (Excludes 1 taking it late.)