September 12, 2003

Homework #2, EECS 206,F03. Due Fri. Sept.19

Notes

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- Reading: Ch. 2 of text and Appendix A of text (complex numbers).
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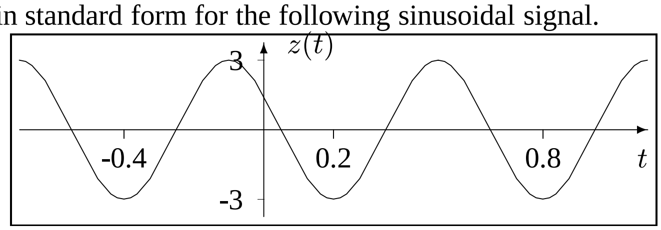
Skill Problems

1. [15] Concept(s): ***correlation and the effect of signal operations***
In the readings, it was stated that $C_N(ax, by) = C_N(x, y)$. This is correct if $ab > 0$, but not quite if $ab < 0$.
Let $x(t)$, $y(t)$, and $z(t)$ denote signals, and let a and b denote nonzero real numbers.
Show the following relationships.
(a) [5] $C(ax, by) = abC(x, y)$
(b) [5] $C(x, y + z) = C(x, y) + C(x, z)$. (These first two properties are called **bilinearity**.)
(c) [0] $C(x, y) = C(y, x)$ (for real signals)
(d) [5] $C_N(ax, by) = \begin{cases} C_N(x, y), & ab > 0 \\ -C_N(x, y), & ab < 0. \end{cases}$
[0] What happens if $ab = 0$?
(e) [0] $C_N(x, \alpha x) = \begin{cases} 1, & \alpha > 0 \\ -1, & \alpha < 0. \end{cases}$ (In fact, $C_N(x, y) = \pm 1$ if and only if y is an amplitude-scaled version of x, so y and x have identical "shapes.")
(Think about how correlation is affected by other signal operations, *e.g.*, amplitude shift.)

2. [25] Concept(s): ***representations of sinusoidal signals***
When relevant below, use the principal value for the phase: $-\pi < \phi \leq \pi$.
(a) [5] A sinusoidal signal $x(t)$ has amplitude=5, frequency 40Hz, and phase = $\pi/3$ radians.
Sketch $x(t)$ carefully by hand, labeling your axes.
(b) [5] Express the following signal in the standard form, *i.e.*, in the form $A\cos(2\pi f_0 t + \phi)$:
$$y(t) = -7\sin(8\pi(t - 3) + 13\pi/4).$$
(c) [5] Find an expression in standard form for the following sinusoidal signal.


(d) [5] Simplify the following sum of sinusoidal signals into standard form:
$$s(t) = 5\sin(8t) + 5\cos(8t - \pi/3).$$
(e) [5] Find a complex-valued signal $\bar{x}(t)$ such that $x(t) = \mathrm{Re}(\bar{x}(t))$, for $x(t)$ as defined in part (a).

3. [30] Concept(s): ***complex arithmetic***
   (a) [15] Convert the following complex numbers from cartesian form to complex exponential form and plot in the complex plane: $z_1 = \sqrt{3} + \jmath$, $z_2 = -\sqrt{2} + \jmath\sqrt{2}$, $z_3 = -2 - \jmath$.
   (b) [10] Determine the product of z_1 and z_2 by:
   - performing multiplication entirely in cartesian coordinates,
   - performing multiplication entirely with the exponential forms of these complex variables.

   (c) [10] Determine the ratio z_1/z_2 by:
   - performing division by first converting z_1 and z_2 to exponential form,
   - performing division by multiplying the numerator and denominator of z_1/z_2 by $z_2^\star$.

   (d) [0] Which form is easier for multiplication and division? What about for addition and subtraction?

4. [30] Concept(s): ***complex arithmetic***
   Simplify the following complex-valued expressions.
   For (a)-(d), give answers in both Cartesian form and exponential (or polar) form.
   (a) [4] $2e^{\jmath\pi/3} + 4e^{-\jmath\pi/6}$
   (b) [4] $(\sqrt{3} - \jmath 3)^9$
   (c) [4] $(\sqrt{3} - \jmath 3)^{-1}$
   (d) [4] $(\sqrt{3} - \jmath 3)^{1/3}$. Hint: there are *three* answers.
   (e) [4] $\mathrm{Re}\left(\jmath\, e^{-\jmath\pi/3}\right)$
   (f) [10] Determine *all* solutions θ (in radians) to the following equation: $\mathrm{Re}\left((1-j)e^{\jmath\theta}\right) = -1$.

5. [15] Concept(s): ***sums of sinusoidal signals with same frequency and phasors, effect of time shift/scale***
   Simplify the following sums of sinusoidal signals into standard form.
   (a) [0] $x_1(t) = 5\cos(2t + \pi/4) + 5\cos(2t + 3\pi/4) - \cos(2t + \pi/2)$.
   Hint. Using phasors, $x_1(t) = (5\sqrt{2} - 1)\cos(2t + \pi/2) \approx 6.071\cos(2t + \pi/2)$.
   (b) [10] $x_2(t) = 5\cos(\pi t + \pi/2) + 5\sin(\pi t - \pi/6) - \cos(\pi t - 2\pi/3)$
   (c) [5] $x_3(t) = x_1(-3(t-2))$, where $x_1(t)$ was defined in (a).

Mastery Problems

6. [20] Concept(s): ***sinusoids and linear systems***
   Sinusoidal signals are particularly important because when a sinusoid is the input to a linear time-invariance (LTI) system, the output is also a sinusoid, and this property is unique to sinusoids!
   Consider a system with an "echo:" the output signal $y(t)$ is the sum of the input signal $x(t)$ and a delayed version of $x(t)$. (You may have experienced something like this in some cell phone calls.) Assume that the following input/output relationship describes the system: $y(t) = x(t) + x(t-1)$.
   (a) [10] If $x(t) = A\cos(2\pi f_1 t)$, show that the output $y(t)$ can be written as $B\cos(2\pi f_2 t + \phi)$.
   Relate B, ϕ and f_2 to A and f_1. This is called the "sine in, sine out" property.
   Hint. Use this "phase splitting" trick: $1 + e^{\jmath\gamma} = e^{-\jmath\gamma/2}\left[e^{\jmath\gamma/2} + e^{-\jmath\gamma/2}\right] = e^{-\jmath\gamma/2} 2\cos(\gamma/2)$.
   (b) [10] Now instead of a sinusoidal input, suppose that the input signal $x(t)$ is periodic with period 4 and $x(t) = 1$ for $0 < t < 2$ and $x(t) = 0$ for $2 < t < 4$. Sketch the input $x(t)$ and the output signal $y(t)$.
   [0] Is there a "square wave in, square wave out" property?

7. [15] Concept(s): ***Euler's formula***
   (a) [10] Prove the following equality, called DeMoivre's formula, using Euler's formula.

$$(\cos\theta + \jmath\sin\theta)^n = \cos(n\theta) + \jmath\sin(n\theta).$$

   (b) [5] Use this result to evaluate $(3 + \jmath 4)^{99}$.