Homework #4, EECS 206, F03. Due **Fri. Oct. 3**

Notes

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- Reading: "Part 3a" lecture notes. Ch. 3 of text, 3.4.5 supplement (on web).

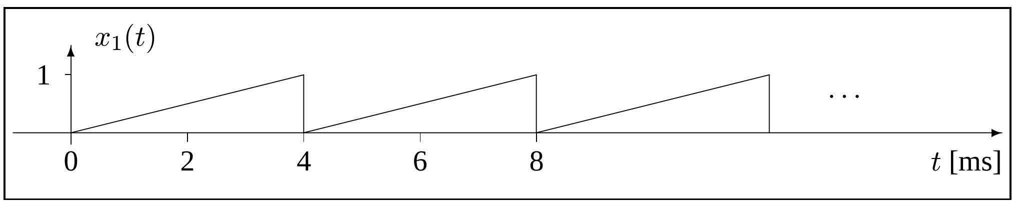
Skill Problems

1. [15] Concept(s): ***AM radio, spectra of sums-of-sinusoids***
Assuming $f_c = 100$kHz and $f_v = 3$kHz, carefully sketch the spectrum of the signal $x(t)$ that is the output of the following system that is similar to an AM radio transmitter.

$$\begin{array}{ccccccc} 8\cos^2(2\pi f_v t) & \rightarrow & \bigoplus & \longrightarrow & \bigotimes & \rightarrow & x(t) \\ & & \uparrow & & \uparrow & & \\ & & 10 & & \cos(2\pi f_c t) & & \end{array}$$

Be sure to label all important features of your sketch.

2. [30] Concept(s): ***Fourier series analysis and spectra of periodic signals***
(a) [0]



Use the Fourier series analysis equation to show that the FS coefficients of the above sawtooth signal are given by:

$$\alpha_k = \begin{cases} 1/2, & k = 0 \\ \frac{1}{2\pi k} e^{\jmath\, \pi/2}, & k \neq 0. \end{cases}$$

Do the integration by hand. Hint. Yes, it does require integration by parts.
You can try to check your work by using MATLAB's symbolic integration, but you may find the results surprising. `pretty(int('t * exp(-i*2*pi*k*t)', 't', 0, 1))`

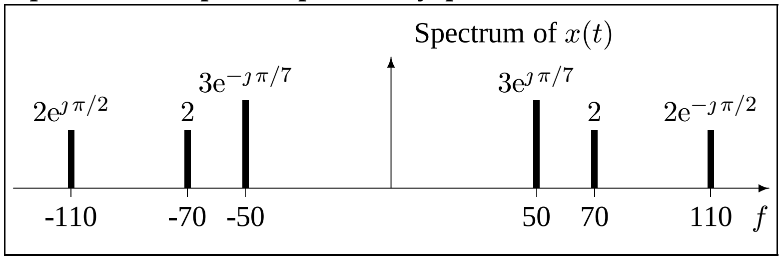
(b) [5] Carefully sketch the spectrum of $x_1(t)$. Use a frequency axis that is wide enough to show several of the components. Hint. There should be no "T" or "k" anywhere in your sketch.

(c) [15] Determine the FS coefficients of the signal $x_2(t) = |\cos(30\pi t)|$ and carefully sketch its spectrum.

(d) [10] Determine the FS coefficients of the signal $x_3(t) = 16\cos^2(2\pi t)$ and carefully sketch its spectrum. Hint. Do you really want to use the analysis formula?

(e) [0] Determine the 10-second Fourier series coefficients $\{\alpha_k\}$ of the signal $x_4(t) = 6\sin(2\pi 5t)$ and carefully sketch its spectrum.

3. [30] Concept(s): ***signal properties from spectra: periodicity, power***



(a) [5] Is this signal periodic? If so, determine its fundamental period.

(b) [5] Determine the DC value of $x(t)$.

(c) [10] Determine the average power of $x(t)$.

(d) [0] Find $x(t)$.

(e) [10] Determine the average power of the signal $x(t) = 7 + 3\cos(4\pi t + 7) + 4\cos(8t - 5)$.
Hint: the very hard way to do this would be to use the formula in the Part 1 lecture notes. Do not do it that way!

Mastery Problems

4. [10] Concept(s): ***Fourier series of even and odd signals***
Show that if $x(t)$ is a periodic signal that is conjugate-symmetric, *i.e.*, $x(-t) = x(t)^\star$, then its Fourier series coefficients are real. Hint. Use the analysis formula to show that $\alpha_k^\star = \alpha_k$.
It follows that for a real periodic signal that is even, *i.e.*, $x(-t) = x(t)$, the FS coefficients are real.
One can also show that for a real periodic signal that is odd, *i.e.*, $x(-t) = -x(t)$, the FS coefficients are imaginary.

5. [15] Concept(s): ***spectra and communication system design***
The amplitude modulation system described in lecture and in the lecture notes is called "double sideband (DSB) modulation," and is the standard for commercial AM radio. An alternative approach is "single sideband (SSB) modulation" and can be described by the following block diagram:

$$\text{Audio: } v(t) \to \underset{\substack{\uparrow \\ d}}{\bigoplus} \longrightarrow \underset{\substack{\uparrow \\ \cos(2\pi f_c t)}}{\bigotimes} \to \boxed{\text{High-pass filter}} \to x(t) \to \text{Antenna}$$

In this system, the "filter" is a circuit that removes all signal components having frequencies that are less than the carrier frequency f_c and greater than $-f_c$. The filter leaves all other frequency components untouched. The purpose of this problem is to reinforce your understanding of spectra and the AM radio application in particular, and to explore the advantages and disadvantages of this alternative modulation scheme.

(a) [10] Suppose that $v(t) = A\cos(2\pi f_v t)$, where $A < d$ and $f_v \ll f_c$. Sketch the spectrum of $x(t)$.
Hint: first sketch the spectrum of the signal that is the *input* to the filter.

(b) [5] Suppose the maximum audio frequency f_v of interest is just a bit less than 5 kHz. If SSB were used by multiple radio stations and you wanted to avoid signal interference, what would be the minimum spacing of the carrier frequencies of the different stations? (Explain.) Hint: the best answer involves a number that is smaller than 10kHz.

(c) [0] In light of the preceding answer, why do you think the double sideband method was chosen (long ago) as the standard for commercial AM radio?
Single sideband *is* used in a variety of communications applications, including NASA's link with the space shuttle: `http://www.teachspace.org/resources/shortwave.html` .