Homework #6, EECS 206, Fall 2003. Due **Friday October 24**

Notes

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- Reading: "Part 3d" lecture notes. (Do not read Chapter 9!)
  As needed: Section 3 in Prof. Wakefield's "Primer on DFT."

Skill Problems

1. [10] Concept(s): ***Inverse DFT***

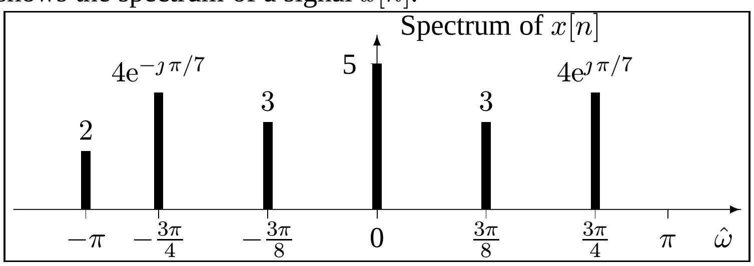
   A signal $x[n]$ has 8-point DFT coefficients given by $X[k] = \begin{cases} 10 - |4-k|, & k \text{ even} \\ 0, & k \text{odd}. \end{cases}$

   (a) [5] Sketch the spectrum of this signal.

   (b) [5] Sketch the signal $x[n]$ from $n = 0, \ldots, 8$. Hint. First express $x[n]$ as a sum-of-sinusoids.

2. [15] Concept(s): ***DFT coefficients, period, RMS, from spectrum***

   The following figure shows the spectrum of a signal $x[n]$.



   (a) [10] Determine the N-point DFT coefficients $X[k]$ of this signal, where N is the fundamental period of $x[n]$. Your answer should be a formula or a list in the form

$$X[k] = \begin{cases} ?, & k = 0 \\ ?, & k = 1 \\ \vdots \end{cases}$$

   (b) [5] Determine the RMS value of the signal $x[n]$ above.

3. [25] Concept(s): ***DFT***

   Determine the N-point DFT of each of the following signals.

   (a) [5] $x_1[n] = \begin{cases} 1, & n = 3 \\ 0, & n = 1, 2, 4, 5, \ldots, N-1 \end{cases}$

   (b) [5] $x_2[n] = 7$ for $n = 0, 1, \ldots, N-1$

   (c) [5] $x_3[n] = \sin(2\pi mn/N)$ for $n = 0, 1, \ldots, N-1$, where $m \in \{0, 1, 2, \ldots, N-1\}$. Hint: use Euler!

   (d) [5] $x_4[n] = \begin{cases} 1, & n \text{ even} \\ 0, & n \text{ odd.} \end{cases}$ (Assume N is even here.) Hint: use $\frac{1+(-1)^n}{2}$ and $-1 = e^{j\pi}$.

   (e) [5] Sketch the spectrum of $x_4[n]$.

4. [10] Concept(s): ***DFT properties***

   Show the following DFT properties using the synthesis equation for the DFT.

   (a) [0] $x[n + mN] = x[n]$ for $m \in \mathbb{Z}$

   (b) [0] If $y[n] = x[n - n_1]$ then $Y[k] = X[k]\, e^{-j\frac{2\pi}{N}kn_1}$.

   (c) [10] Suppose a N-periodic signal $x[n]$ has DFT $X[k] = 2^k$, $k = 0, \ldots, N-1$.
   Determine the DFT of the signal $y[n] = 7 + 3x[n-5]$.

Mastery Problems

5. [10] Concept(s): ***DFT properties***

Consider the 8-periodic discrete-time sawtooth signal $x[n]$ described by $x[n] = \begin{cases} 10n, & n = 0, \ldots, 7 \\ \text{periodic}, & \text{otherwise.} \end{cases}$

One can show that the 8-point DFT of this signal is given by $X[k] = \dfrac{10}{e^{-\jmath(\pi/4)k} - 1}$.

Without using the DFT synthesis formula or MATLAB, sketch (for $n = 0, \ldots, 7$) the signal $y[n]$ whose 8-point DFT is given by $Y[k] = \dfrac{5e^{-\jmath(\pi/2)k}}{e^{-\jmath(\pi/4)k} - 1}$.

6. [30] Concept(s): ***Use of DFT for signal compression***

In lecture, we demonstrated that one could perform audio signal compression by computing the DFT of a signal, discarding the "small" DFT coefficients, and then reconstructing (a close approximation to) the original signal using the inverse DFT (the synthesis formula). This is the essence of how MP3 digital audio compression and JPEG digital image compression work. The purpose of this problem is to give you firsthand experience with these signal compression methods.

(a) [0] Download the MATLAB file `dft_chop.m` from the homework page of the course web site and run it. Do this right away to avoid last minute web server / MATLAB problems!

(b) [10] Modify the program so that it computes and prints the mean squared error (MSE) between the original signal $x[n]$ and the signal $y[n]$ that is reconstructed from the "large" DFT coefficients.

(c) [0] The program includes a user-selectable "threshold" that determines what is a "small" DFT coefficient that should be discarded. Explore various values for this threshold, and observe the effect on the MSE.

(d) [20] Determine the largest possible threshold that satisfies the accuracy design criterion $\mathrm{MSE}(x, y) < 0.0018$. You can do this by trial and error (to a modest number of decimal places), or you may write a loop to do this "automatically." (Either way, include a printout of your modified program.)

(e) [0] For the threshold you found, how much is the data storage reduced?

7. [10] Concept(s): ***relating spectra of continuous-time and discrete-time signals***

The continuous-time signal $x(t) = 3\sin(2\pi 100t) + 6\cos(2\pi 200t) + 2\sin(2\pi 600t)$ is sampled with sampling interval $T_s = 2.5$ms to yield a discrete-time signal $x[n]$.

(a) [0] Carefully sketch the two-sided spectrum of $x(t)$.

(b) [10] Carefully sketch the two-sided spectrum of $x[n]$.

(c) [0] What happened?

Optional Extra Credit Problem

8. [10] (Optional extra credit challenge problem. No help will be given in office hours for such problems.)

Suppose $x[n]$ is N-periodic and has Fourier series coefficients $X[k]$. Define a new signal $y[n] = \sin(2\pi \frac{m}{N} n) x[n]$, and let $Y[k]$ denote its Fourier series coefficients. Determine how to express each $Y[k]$ in terms of the $X[k]$'s.