Homework #7, EECS 206, Fall 2003. Due **Friday October 31**

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- Reading: Chapter 4, "Part 4" lecture notes.

Skill Problems

1. [5] Concept(s): ***quantization***
   An A/D converter is designed to work with signals with values ranging from -5V to +5V.
   For a certain application, the maximum quantization error that is acceptable is 0.1V. Determine the minimum number of bits per signal sample that can be used in the A/D converter and still meet this accuracy specification.

2. [20] Concept(s): ***signal interpolation/reconstruction in D/A conversion***
   The purpose of this problem is to compare *quantitatively* the two simplest signal interpolation methods that are used in D/A conversion: zero-order hold, and linear interpolation. Consider the following scenario:

$$x(t) \to \boxed{\text{C-to-D}} \to x[n] \to \boxed{\text{D-to-C}} \to \hat{x}(t)\,.$$

   A good interpolator should have $\hat{x}(t) \approx x(t)$, so $\mathrm{MSD}(\hat{x} - x)$ should be small. In this problem we consider the following 16-periodic signal: $x(t) = \begin{cases} 1, & 0 \le t < 3 \\ 0, & 3 \le t < 16 \\ \text{periodic} & \text{otherwise}. \end{cases}$

   (a) [5] Assume the sampling interval is $T_\mathrm{s} = 4$. Sketch the samples $x[n]$ for $n = 0, 1, \ldots, 8$.

   (b) [5] Let $\hat{x}_1(t)$ denote the result of zero-order hold signal interpolation from $x[n]$. Sketch both $x(t)$ and $\hat{x}_1(t)$ on the same graph for $0 \le t \le 32$. (Use two colors.) Does $\hat{x}_1(t)$ look like a good approximation to $x(t)$?

   (c) [5] Let $\hat{x}_2(t)$ denote the result of linear interpolation from $x[n]$. Sketch both $x(t)$ and $\hat{x}_2(t)$ on the same graph for $0 \le t \le 40$. Does $\hat{x}_2(t)$ look like a better or worse approximation to $x(t)$ than $\hat{x}_1(t)$ did?

   (d) [0] Determine $\mathrm{MSD}(x, \hat{x}_1)$ and $\mathrm{MSD}(x, \hat{x}_2)$ using time-domain calculations. Quantitatively, for this particular signal $x(t)$, which interpolation method is better and why?

   (e) [0] Suppose that instead we used the ideal sinc interpolator: $\hat{x}(t) = \sum_n x[n]\, p(t - nT_\mathrm{s})$, where $p(t) = \frac{\sin(\pi t/T_\mathrm{s})}{\pi t/T_\mathrm{s}}$. Would $\mathrm{MSD}(x, \hat{x}) = 0$, *i.e.*, would $\hat{x} = x$ in this case? Why or why not?

   (f) [5] Suppose instead that the input signal was $x(t) = \cos(2\pi \frac{1}{10} t) + \cos(2\pi \frac{1}{9} t)$.
   Determine $\mathrm{MSD}(x, \hat{x})$ when $p(t) = \frac{\sin(\pi t/T_\mathrm{s})}{\pi t/T_\mathrm{s}}$. Hint. Do not do any integration.

3. [20] Concept(s): ***aliasing of sinusoidal signals***

A continuous-time sinusoidal signal is sampled at the sampling rate $f_s = 3000$Hz, yielding the following discrete-time signal:

$$x[n] = 10\cos\left(\frac{3}{4}\pi n + \frac{\pi}{5}\right).$$

Give *four* examples of continuous-time signals that could have been the original signal:

(a) [5] A sinusoid $x_0(t)$ corresponding to an unaliased case.

(b) [5] A sinusoid $x_1(t)$ corresponding to "case 1" aliasing (see lecture notes).

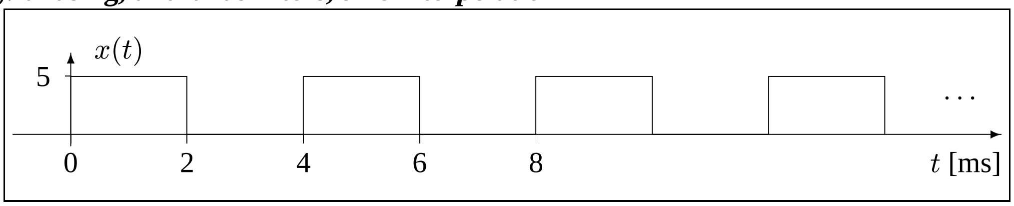
(c) [5] A sinusoid $x_2(t)$ corresponding to "case 2" aliasing (folding).

(d) [5] A signal $x_3(t)$ that is *not* a sinusoid.

For the sinusoids, just give a formula. For $x_3(t)$, sketch a picture.

Mastery Problems

4. [20] Concept(s): ***aliasing, anti-alias filters, sinc interpolation***



The signal $x(t)$ above is passed through a perfect anti-alias filter (that removes all frequency components at or above the frequency $f_s/2$), is sampled, and then interpolated using the ideal sinc interpolator as follows:

$$x(t) \to \boxed{\text{Perfect anti-alias}} \to x_a(t) \to \boxed{\text{Sample at rate } f_s} \to x[n] \to \boxed{\text{Ideal sinc interpolator}} \to \tilde{x}(t)$$

Determine $\tilde{x}(t)$ when $f_s = 600$Hz.

Hint. First find the spectrum of $x(t)$ and then eliminate most of the components to find $x_a(t)$.

Optional Problems

5. [20] (Optional extra credit challenge problem. No help will be given in office hours for such problems.)

Back in the 20th century, engineers used mathematical reference books containing useful formulas such as

$$\sum_{k=1}^{\infty}\frac{1}{k^2} = \frac{\pi^2}{6}, \qquad \text{and} \qquad \sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k^2} = \frac{\pi^2}{12}, \qquad \text{and} \qquad \sum_{l=0}^{\infty}\frac{1}{(2l+1)^2} = \sum_{\substack{k=1\\k \text{ odd}}}^{\infty}\frac{1}{k^2} = \frac{\pi^2}{8}.$$

In the 21st century, one can find such formulas at web sites like `http://mathworld.wolfram.com`. Have you ever wondered how such formulas were discovered in the first place? Interestingly, all of the above formulas can be derived using concepts covered in EECS 206 so far.

For extra credit, derive the following formula using concepts from EECS 206

$$\sum_{\substack{k=1\\k \text{ odd}}}^{\infty}\frac{1}{k^4} = \frac{\pi^4}{96}.$$

Obscure hint. Think about triangles.