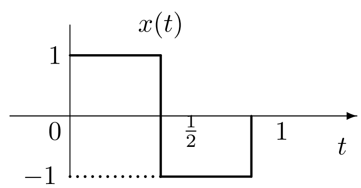
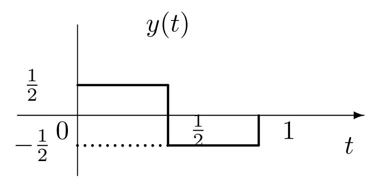
		Answer
(1)	(a) (b) (c) (d) (e)	(d)
(2)	(a) (b) (c) (d) (e)	(b)
(3)	(a) (b) (c) (d) (e)	(c)
(4)	(a) (b) (c) (d) (e)	(d)
(5)	(a) (b) (c) (d) (e)	(d)
(6)	(a) (b) (c) (d) (e)	(e)
(7)	(a) (b) (c) (d) (e)	(b)
(8)	(a) (b) (c) (d) (e)	(d)
(9)	(a) (b) (c) (d) (e)	(a)
(10)	(a) (b) (c) (d) (e)	(c)
Subtotal		
(11)		$8.6972\cos(\omega_0 t - 2.85)$
(12)		$C_0 = 0$ and $C_k = j/k$ for $k \neq 0$
Total		

(1) The normalized correlation $C_N(x,y)$ of the following two signals with the support interval $[0,1]$ is

(a) -1
(b) $-\frac{1}{\sqrt{2}}$
(c) $\frac{1}{\sqrt{2}}$
(d) 1
(e) None of the above





Note that

$$C_N(x,y) = \frac{\displaystyle\int_0^1 x(t)y^*(t)\,dt}{\sqrt{\displaystyle\int_0^1 |x(t)|^2\,dt}\sqrt{\displaystyle\int_0^1 |y(t)|^2\,dt}},$$

where

$$\int_0^1 x(t)y^*(t)\,dt = \int_0^{1/2} 1\frac{1}{2}\,dt + \int_{1/2}^1 (-1)\cdot(-\frac{1}{2})\,dt = \frac{1}{2},$$

$$\int_0^1 |x(t)|^2\,dt = \int_0^{1/2} 1\,dt + \int_{1/2}^1 1\,dt = 1,$$

$$\int_0^1 |y(t)|^2\,dt = \int_0^{1/2} |\frac{1}{2}|^2\,dt + \int_{1/2}^1 |-\frac{1}{2}|^2\,dt = \frac{1}{4}.$$

Therefore, $C_N(x,y) = 1$.

(Alternative approach) Note that $y(t) = \frac{1}{2}x(t)$. Then they are positively (completely) correlated.

The answer is (d).

(2) The average power of $x(t) = Ae^{j(\omega_0 t+\phi)}$ over the interval $[0, T_0]$, where $A > 0$ and $\omega_0 = \frac{2\pi}{T_0}$, is

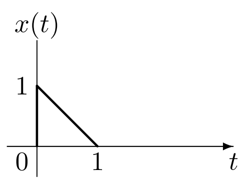
(a) $A^2 \cos^2 \phi$
(b) A^2
(c) $A^2 T_0$
(d) $A^2 |\cos \phi|$
(e) None of the above

The average power

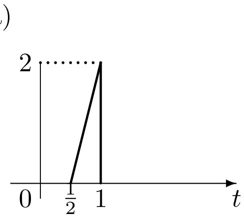
$$\frac{1}{T_0}\int_0^{T_0} |x(t)|^2\, dt = \frac{1}{T_0}\int_0^{T_0} Ae^{j(\omega_0 t+\phi)}(Ae^{j(\omega_0 t+\phi)})^*\, dt = \frac{1}{T_0}\int_0^{T_0} Ae^{j(\omega_0 t+\phi)}Ae^{-j(\omega_0 t+\phi)}\, dt$$
$$= \frac{1}{T_0}\int_0^{T_0} A^2\, dt = A^2.$$

The answer is (b).

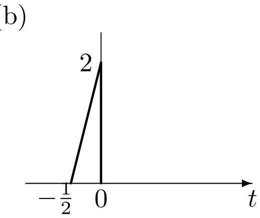
(3) For signal $x(t)$ given below, the signal $y(t) = 2x\big(-\frac{1}{2}t + 1\big)$ looks like



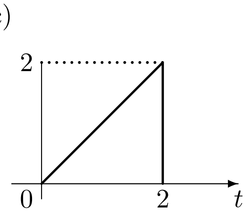
(a)



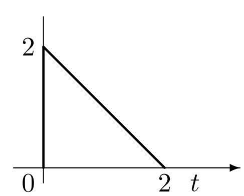
(b)



(c)



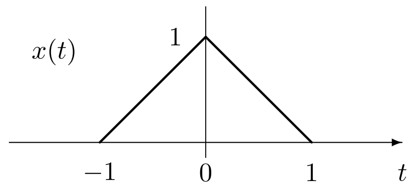
(d)



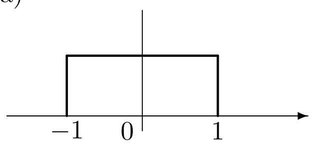
(e) None of the above

The answer is (c).

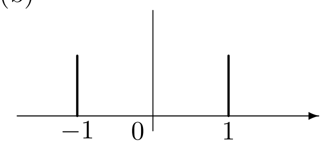
(4) The distribution of signal values of $x(t)$ over the support interval $[-1, 1]$ looks like



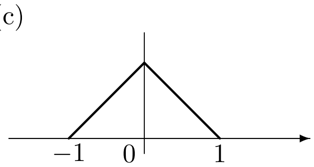
(a)



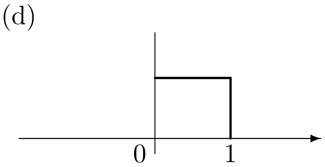
(b)



(c)



(d)

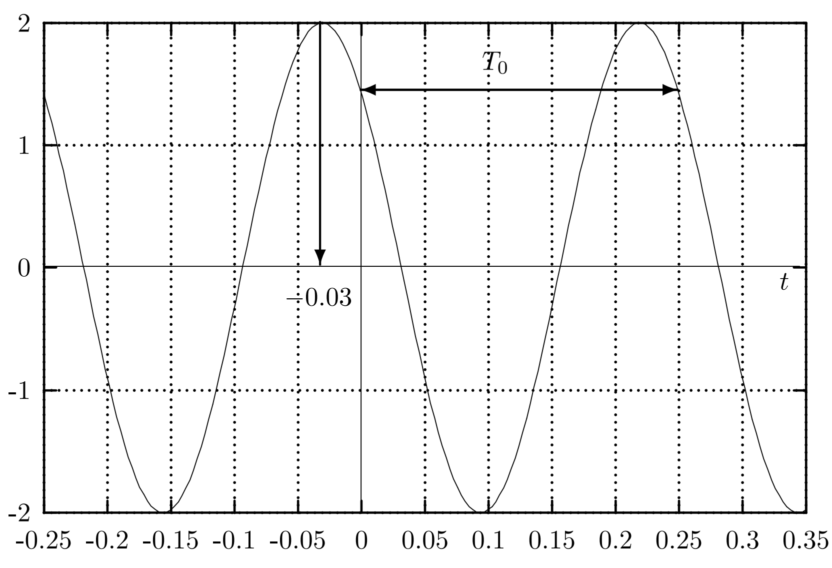


(e) None of the above

Note that the signal has the maximum value of 1 and the minimum of 0.

The answer is (d).

(5) The following graph of a signal is best described by



(a) $2\cos(2\pi 2t - \frac{\pi}{4})$
(b) $2\cos(2\pi 4t - \frac{\pi}{4})$
(c) $2\cos(2\pi 2t + \frac{\pi}{4})$
(d) $2\cos(2\pi 4t + \frac{\pi}{4})$
(e) $2\cos(2\pi 4t + \frac{\pi}{10})$

Let $x(t) = A\cos(\omega_0 t + \phi) = A\cos(\omega_0(t + \phi/\omega_0))$. Then $A = 2$.

First, the period T_0 is found, for example, from observing the same pattern $t_1 = 0$ and $t_2 = 0.25$. Then $T_0 = t_2 - t_1 = 0.25$ or $\omega_0 = \frac{2\pi}{T_0} = 2\pi 4$ rad/s. Secondly, the time at which the first negative maximum occurs is about $-0.03 \implies \phi/\omega_0 \approx 0.03$. Or $\phi \approx \omega_0 0.03 = 8\pi \cdot 0.03 = 0.24\pi$. The closest phase that is given is $\frac{\pi}{4} = 0.25\pi$.

The answer is (d).

(6) The signal

$$\Re\{3e^{j(6t+\frac{\pi}{3})} + j4e^{j(6t+\frac{\pi}{2})}\}$$

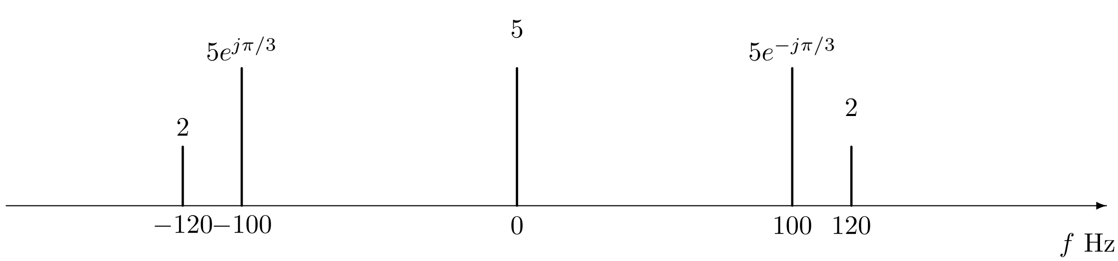
equals

(a) $2\sqrt{2}\cos(6t - \frac{\pi}{4})$
(b) $2\sqrt{2}\cos(6t + \frac{\pi}{4})$
(c) $2\sqrt{2}\cos(6t - \frac{3\pi}{4})$
(d) $2\sqrt{2}\cos(6t + \frac{3\pi}{4})$
(e) None of the above

$$\begin{aligned}\Re\{3e^{j(6t+\frac{\pi}{3})} + j4e^{j(6t+\frac{\pi}{2})}\} &= \Re\{e^{j6t}(3e^{j\frac{\pi}{3}} + j4e^{j\frac{\pi}{2}})\}\\ &= \Re\{e^{j6t}(3e^{j\frac{\pi}{3}} + j4\cdot j)\} = \Re\{e^{j6t}(3\cos(\frac{\pi}{3}) + j3\sin(\frac{\pi}{3}) - 4)\}\\ &= \Re\{e^{j6t}3.6056e^{j2.3370}\} = 3.6056\cos(6t + 2.3370).\end{aligned}$$

The answer is (e).

(7) The spectrum below describes signal



(a) $5 + 5\cos(2\pi 100t - \frac{\pi}{3}) + 2\cos(2\pi 120t)$
(b) $5 + 10\cos(2\pi 100t - \frac{\pi}{3}) + 4\cos(2\pi 120t)$
(c) $5 + 5\sin(2\pi 100t - \frac{\pi}{3}) + 2\sin(2\pi 120t)$
(d) $5 + 10\sin(2\pi 100t - \frac{\pi}{3}) + 4\sin(2\pi 120t)$
(e) None of the above

The signal is, by collecting all the spectral components,

$$5 + 5e^{-j\pi/3}e^{j2\pi 100t} + 2e^{j2\pi 120t} + 5e^{j\pi/3}e^{-j2\pi 100t} + 2e^{-j2\pi 120t} = 5 + 10\cos(2\pi 100t - \frac{\pi}{3}) + 4\cos(2\pi 120t).$$

The answer is (b).

(8) A real signal $x(t)$ with the following spectrum

$$C_0 = 10, \quad C_1 = 5e^{j\pi/2}, \quad C_2 = 1e^{j\pi},$$

and $C_k = 0$ for $k \geq 3$, has the average power of

(a) 16
(b) 100
(c) 126
(d) 152
(e) None of the above

The power is given by

$$\sum_{k=-\infty}^{\infty} |C_k|^2 = C_0^2 + |C_1|^2 + |C_2|^2 + |C_{-1}|^2 + |C_{-2}|^2 = 152.$$

The answer is (d).

(9) Given a periodic signal $x(t)$ with the fundamental period T_0 and $y(t) = e^{j2\pi \frac{k}{T_0} t}$, the unnormalized correlation $C(x, y)$ over the interval $[-\frac{T_0}{2}, \frac{T_0}{2}]$ is given by

(a) $C_k T_0$
(b) $C_k T_0/2$
(c) $|C_k| T_0$
(d) $|C_k| T_0/2$
(e) Insufficient information given

where C_k is the Fourier coefficient of signal $x(t)$ at frequency $\frac{k}{T_0}$ Hz.

Note that

$$\begin{aligned} C_k &= \frac{1}{T_0} \int_{T_0} x(t) e^{-j2\pi kt/T_0} \, dt \\ &= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) e^{-j2\pi kt/T_0} \, dt = \frac{1}{T_0} c(x, y). \end{aligned}$$

So,

$$c(x, y) = \int_{T_0} x(t) e^{-j2\pi kt/T_0} \, dt = C_k T_0.$$

The answer is (a).

(10) For a real signal with the Fourier coefficient $C_{-4} = 5e^{j\frac{\pi}{2}}$, the value of C_4 is

(a) $5e^{j\frac{\pi}{2}}$
(b) $-5e^{j\frac{\pi}{2}}$
(c) $5e^{-j\frac{\pi}{2}}$
(d) $-5e^{-j\frac{\pi}{2}}$
(e) Insufficient information given

For a real signal, $C_{-k} = C_k^*$. Therefore, $C_k = C_{-k}^*$. Then $C_4 = C_{-4}^* = 5e^{-j\frac{\pi}{2}}$.

The answer is (c).

(11) (15 points) Simplify the following signal in the standard sinusoidal form.

$$x(t) = -4\cos(\omega_0 t) + 5\sin(\omega_0 t - \frac{\pi}{3}).$$

Note that the two component signals are sinusoids of the same frequency and hence we can use the phasor for the reduction.

$$-4\cos(\omega_0 t) + 5\sin(\omega_0 t - \frac{\pi}{3}) = 4\cos(\omega_0 t + \pi) + 5\cos(\omega_0 t - \frac{\pi}{3} - \frac{\pi}{2}).$$

Then the phasor sum is

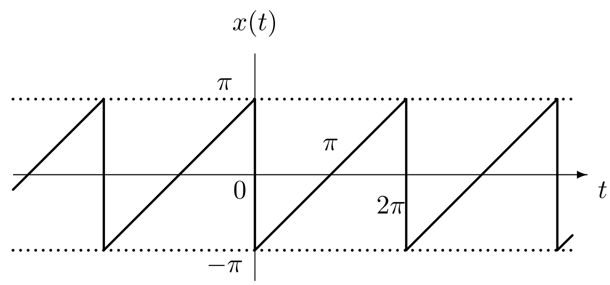
$$4e^{j\pi} + 5e^{-j5\pi/6} = -8.3301 - j2.5000 = 8.6972e^{-j2.8500}.$$

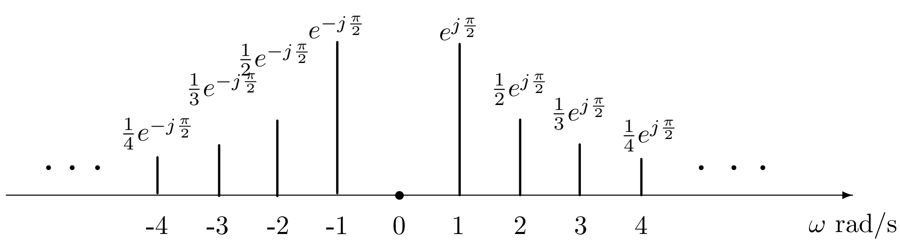
The corresponding sinusoid then is

$$8.6972\cos(\omega_0 t - 2.85).$$

(12) (20 points) Find and plot the (two-sided) spectrum of the following periodic signal

$$x(t) = t - \pi, \quad 0 \le t \le 2\pi.$$





Note that $\int (at+b)e^{ct}\,dt = (at+b)e^{ct}/c - ae^{ct}/c^2 + K$.

Note that $C_0 = 0$, the average over one perioid $[0, 2\pi]$ is zero by inspection. Now C_k for $k \neq 0$,

$$\begin{aligned}
C_k &= \frac{1}{2\pi}\int_0^{2\pi}(t-\pi)e^{-j2\pi\frac{k}{2\pi}t}\,dt = \frac{1}{2\pi}\int_0^{2\pi}(t-\pi)e^{-jkt}\,dt \\
&= \frac{1}{2\pi}\left[\frac{(t-\pi)e^{-jkt}}{-jk} - \frac{e^{-jkt}}{(-jk)^2}\right]_0^{2\pi} \\
&= \frac{1}{2\pi}\left[\frac{(t-\pi)e^{-jkt}}{-jk} + \frac{e^{-jkt}}{k^2}\right]_0^{2\pi} \\
&= \frac{1}{2\pi}\left(\left[\frac{\pi e^{-j2\pi k}}{-jk} + \frac{e^{-j2\pi k}}{k^2}\right] - \left[\frac{-\pi}{-jk} + \frac{1}{k^2}\right]\right) \\
&= \frac{1}{2\pi}\left(\left[\frac{\pi\cdot 1}{-jk} + \frac{1}{k^2}\right] - \left[\frac{\pi}{jk} + \frac{1}{k^2}\right]\right) \\
&= \frac{j}{k}.
\end{aligned}$$