Relevant Reading: Chapter 3: 3.1 to 3.4 (including 3.4.5 handout), 4.1 (up to the end of 4.1.1), 9.1.1
Relevant Items in the DSP First CD: None this week.

1. It can be shown (you do not have to prove this) that if periodic signal $y(t)$ is obtained from periodic signal $x(t)$ by time-shifting, namely,

$$y(t) = x(t - t_d)$$

then the Fourier coefficients C_k^y of $y(t)$ are related to the Fourier coefficients C_k^x of $x(t)$ as follows:

$$C_k^y = C_k^x e^{-jk\omega_0 t_d}.$$

In words, time-shifting in the time domain corresponds to a phase-shift in the frequency domain.

Use this result and the results obtained for the "symmetric square wave" considered in class to show that the trigonometric Fourier series of the square wave in equation (3.4.4) on p. 63 in the textbook is:

$$\sum_{\substack{k=1 \\ k \text{ odd}}} \frac{4}{\pi k} \sin(k\omega_0 t).$$

2. Let C_k be the Fourier coefficients of periodic signal $x(t)$, where the fundamental period of $x(t)$ is T_0. Suppose we calculate a new set of Fourier coefficients for $x(t)$, but this time using period $2T_0$ instead of T_0, namely we calculate C_k^{new} as follows:

$$C_k^{\text{new}} = \frac{1}{2T_0} \int_0^{2T_0} x(t) e^{-jk\frac{2\pi}{2T_0}t}\, dt.$$

Derive an expression for C_k^{new} in terms of C_k.

3. For each of the following discrete-time signals, determine if the signal is periodic and if so, find its fundamental period N_0.

   (a) $x_1[n] = \cos(\frac{\pi}{4}n)$

   (b) $x_2[n] = \cos(\frac{3\pi}{8}n)$

   (c) $x_3[n] = \cos(n)$

   (d) Explain your answers in (a) and (b) in view of the fact that the frequency of $x_2[n]$ is larger than that of $x_1[n]$. Hint: You may find it helpful to plot these signals in Matlab. However, find the periods analytically.

4. Textbook, Problem 4.1, p. 112.