Relevant Reading:
DLN Classnote dated June 3, 2002.
G. Wakefield's "Primer on DFT": Section 3.

1. Calculate the N-point DFTs of the following signals:
   (a) $s[n] = 1$ for $n = 0$, and $s[n] = 0$ for $1 \le n \le N-1$.
   (b) $s[n] = 1$ for $0 \le n \le N-1$.
   (c) $s[n] = \sin(2\pi mn/N)$ for $0 \le n \le N-1$, where m is a fixed integer less than $N/2$. Hint: use the inverse Euler formula.

2. Suppose that the DFT of a periodic signal $x[n]$ is $X[0] = 1$ and $X[k] = 0$ for all other values of k. What is $x[n]$?

3. Suppose that we have a real signal $x[n]$, with fundamental period N_0, whose DFT coefficients are all equal to 0 except $X[k]$ and $X[N_0 - k]$. Let $X[k] = Ae^{j\phi}$.
   (a) What is the average value of $x[n]$?
   (b) Find $x[n]$. Hint: Find $X[N_0 - k]$ first.
   (c) What is the average power of $x[n]$?

4. Consider the synthesis equation of the DFT:
$$x[n] = \sum_{k=0}^{N_0-1} X[k]e^{j2\pi kn/N_0}.$$
   Use that equation to show that $x[n - N_0] = x[n]$.

5. Prove the time-shift DFT property:
$$y[n] = x[n - n_1] \implies Y[k] = e^{-j2\pi kn_1/N_0}X[k].$$

6. Calculate the 4-point DFTs of the following 4-point signals:
   (a) $x[n] = \cos(\pi n/2)$, $n = 0, 1, 2, 3$.
   (b) $x[n] = 2^n$, $n = 0, 1, 2, 3$.

7. Consider the discrete-time square wave of fundamental period N_0, where it is assumed that N_0 is even:
$$x[n] = \begin{cases} 1, & 0 \le n \le \frac{N_0}{2} - 1, \\ 0, & \frac{N_0}{2} \le n \le N_0 - 1. \end{cases}$$
   (a) Calculate $X[0]$.
   (b) Verify that, for $k \ne 0$,
$$X[k] = \frac{1}{N_0} e^{-j\pi \frac{k}{N_0}(\frac{N_0}{2}-1)} \frac{\sin(\pi k/2)}{\sin(\pi k/N_0)}.$$
   Hint: After writing the DFT-analysis equation, use the fact below about geometric series and then factor the first term in the given expression for $X[k]$; the sin terms should "appear"...
$$\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}, \quad a \ne 1.$$