Topics: Systems and Filters

1. Concept of Systems
2. Filters
3. Properties of filters
   (a) Linearity
   (b) Time-invariance
   (c) Causality
   (d) Stability

1 Signals and Systems

- **Signals and Systems**: Lectures mostly about signals so far
- We now study certain kind of systems called *filters*, in particular *linear time-invariant* (LTI) filters.
- Lab 6 experiments with filters, mostly LTI, but also a little with nonlinear filters—median filters.
- We concentrate on discrete-time systems and filters.
  (a) There are also continuous-time systems and filters.
  (b) But we can also do continuous-time filtering by sampling, discrete-time filtering and reconstruction.
  (c) EECS 306 deals with continuous-time systems and filters.

2 Systems

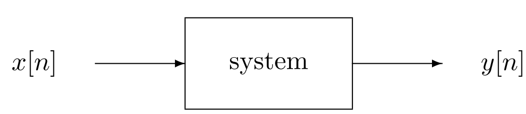
(1) What is a system?

A discrete-time system is an *object* that

(a) takes a discrete-time input signal $x[n]$

(b) and produces a discrete-time output signal (response) $y[n]$ determined by the input signal.

(2) Block diagram



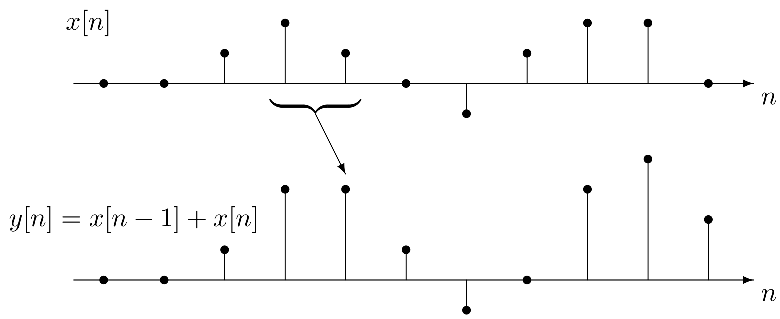
(3) **Notations**

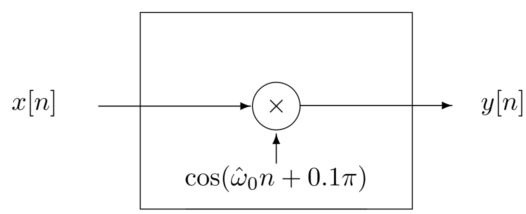
(i) $x[n] \mapsto y[n]$ means that $x[n]$ is the input and $y(t)$ is the response of the system to $x[n]$.

(ii) $y[n] = \mathcal{T}\{x[n]\}$
$\mathcal{T}\{\cdot\}$ is an operator that maps a signal into a signal.

(4) Examples of systems

$$
\begin{aligned}
y[n] &= x[n-1] + x[n] \\
y[n] &= x[n] + 3 \\
y[n] &= x^3[n-1] + 2x[n] \\
y[n] &= x[-n] \\
y[n] &= x[n]\cos(\hat{\omega}_0 n + 0.1\pi)
\end{aligned}
$$

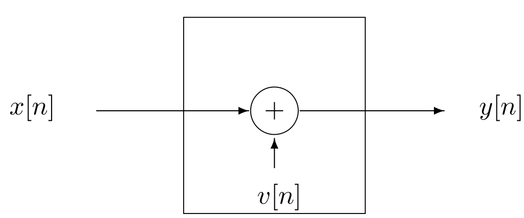




(5) (Deterministic and nondeterministic)

(i) Our systems are *deterministic*, meaning that the output signals is entirely determined by the input signal.

(ii) An example of a *nondeterministic* system



where $v[n]$ is a noise that is not known in advance and not repeatable, e.g., random.

(6) We focus on a special kind of systems called *linear time-invariant* filters.

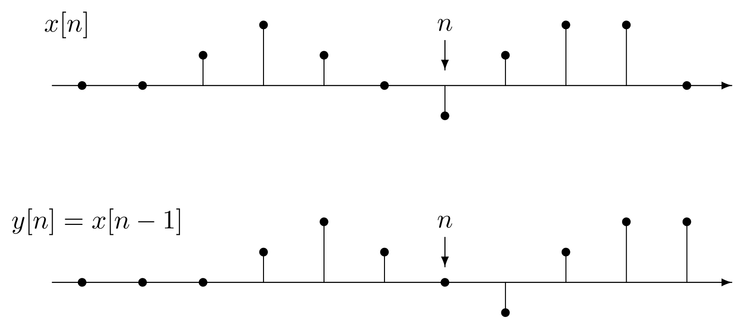
3 Linear Time-Invariant Filters

(1) Linear time-invariant (LTI) filters are very common.

(2) Examples of LTI filters

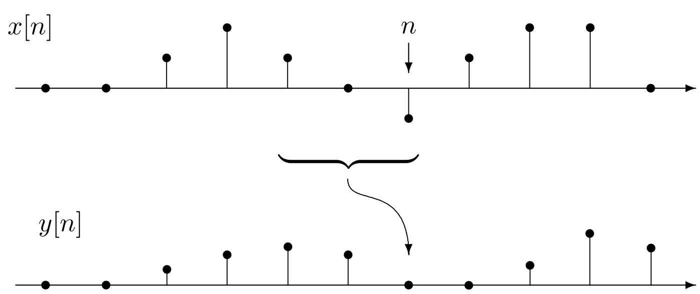
(a) (Delay)

$$y[n] = x[n-1]$$



(b) (Running/Moving/Sliding Average) smoothing

$$y[n] = \frac{1}{3}\big(x[n-2] + x[n-1] + x[n]\big)$$



(General form of Running Average) the larger M, the smoother

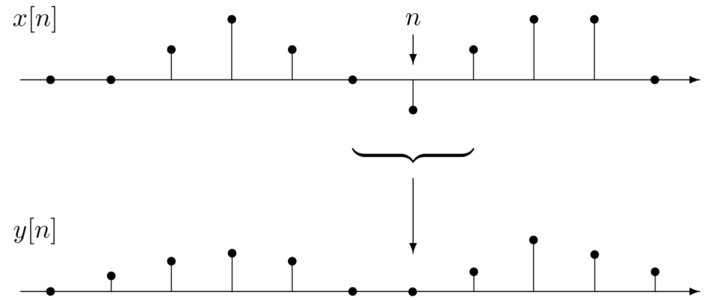
$$y[n] = \frac{1}{M+1}\big(x[n-M] + x[n-M+1] + \cdots + x[n]\big)$$

(c) (Running Sum) smoothes and amplifies

$$y[n] = x[n-2] + x[n-1] + x[n]$$

(d) (Running Average with look ahead)

$$y[n] = \frac{1}{3}\big(x[n-1] + x[n] + x[n+1]\big)$$



(i) Running average with look ahead

(ii) Cannot do this in real time

(iii) Can do this on recorded data

(iv) (Causality) Filters that operate in real time are called *causal*.

The output value at time n depends only on the input values at time n and before, i.e., does *not* depend on the input values of the future time $x[n+1], x[n+2], \ldots$.

(e) (Weighted running average)

$$y[n] = \frac{1}{4}x[n-1] + \frac{1}{4}x[n] + \frac{1}{2}x[n+1])$$

Different choices of weights are for different purposes.

$\boxed{\text{Filters given above have finitely many terms involving the input signal } x[n]. \text{ They are called finite-impulse-response (FIR) filters.}}$

(f) (IIR Filters) The following are not FIR, by infinite-impulse-response (IIR) filters.

(i)

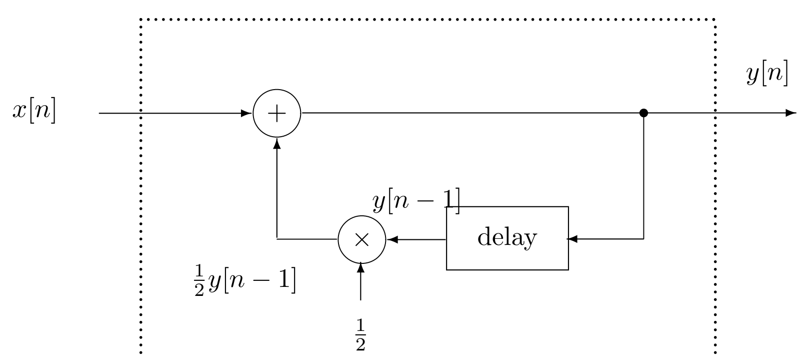
$$y[n] = \frac{1}{2}x[n] + \frac{1}{4}x[n-1] + \frac{1}{8}x[n-2] + \cdots .$$

Note the infinite number of terms involving $x[n]$.

(ii)

$$y[n] = \underbrace{\frac{1}{2}y[n-1]}_{\text{feedback}} + x[n].$$

This turns out to be the same as (i).



General Form of LTI Filters:

$$y[n] = \sum_{k=-M_1}^{M_2} b_k x[n-k],$$

where M_1 and M_2 are integers

$$0 \leq M_1, M_2 \leq \infty.$$

Classification:

1. (FIR or IIR)
   (i) FIR: $M_1, M_2 < \infty$.
   (ii) IIR: M_1 or $M_2 = \infty$.
2. (Causal or noncausal)
   (i) Causal: $M_1 = 0$.
   (ii) Noncausal: $M_1 > 0$.