Topics: Filters

1. LTI filter description
2. Goals of filtering

1 LTI Filters

General Form of LTI Filters:

(a) Filter Description: $\{b_k\}$

(b) Input-output relationship by the difference equation:

$$y[n] = \sum_{k=-M_1}^{M_2} b_k x[n-k],$$

where M_1 and M_2 are integers

$$0 \leq M_1, M_2 \leq \infty.$$

$M_1 + M_2$ are called the *order* of the filter.

Classification:

(a) FIR or IIR

- (i) FIR: $M_1, M_2 < \infty$.
- (ii) IIR: M_1 or $M_2 = \infty$.

(b) Causal or noncausal

- (i) Causal: $M_1 = 0$.
- (ii) Noncausal: $M_1 > 0$.

2 Principal Goals of Filtering

(a) Signal recovery/Noise reduction

(b) Spectral shaping

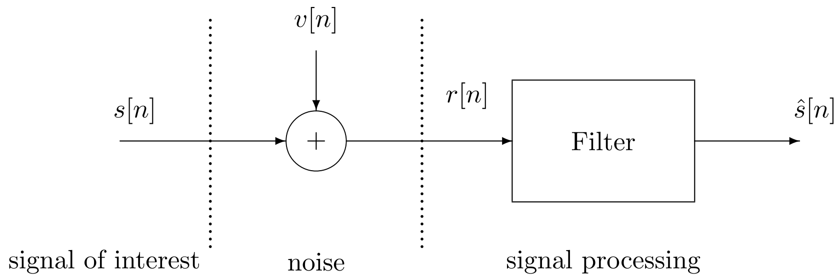
(c) Inverse filtering

(d) Signal detection

We focus on signal recovery and spectral shaping.

2.1 Signal Recovery

Scenario

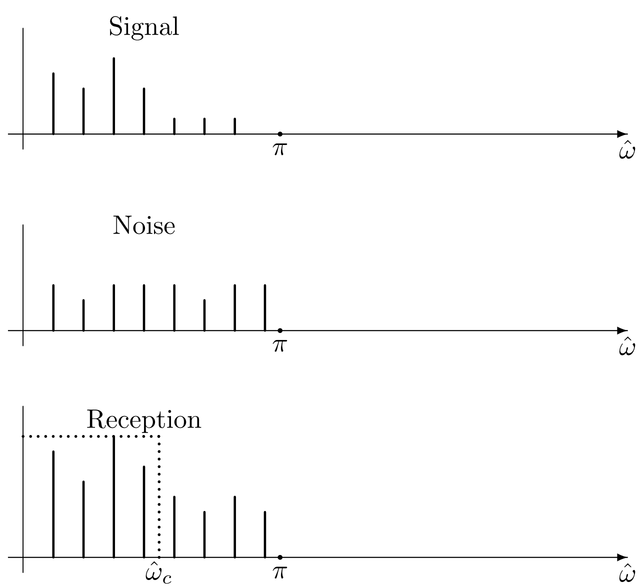


(a) (Goal) want $\hat{s}[n]$ to be $s[n]$, i.e., recover $s[n]$

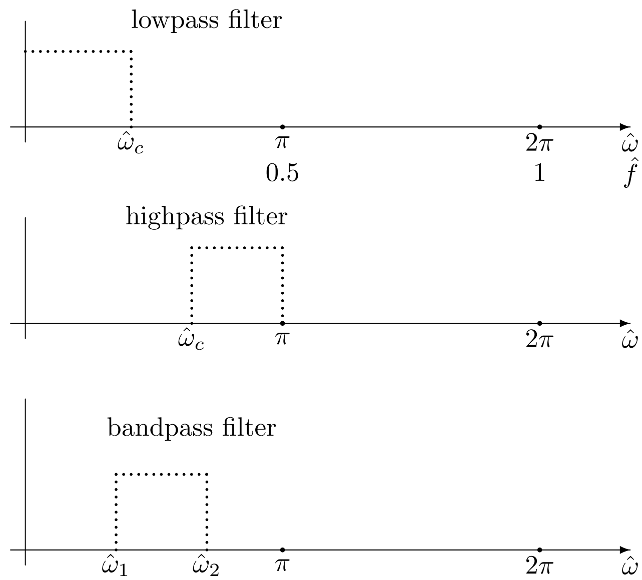
Or want to reduce $v[n]$ as much as possible

(b) (Environment or assumption)

- $s[n]$ and $v[n]$ must differ in characteristics.
- Usually, they differ in spectrum, i.e., distribution of power (energy) in frequency.
- (Example)
  (i) Suppose $v[n]$ has larger spectral values where $s[n]$ has small spectral values



  (ii) Design a filter to block/reject/attenuate at frequencies where noise is much stronger than the desired signal
- Blocking filters



(i) Lowpass filters: block all frequencies above some *cutoff* frequency.
Example: knock out high frequency static/hiss/snow type of noise.

(ii) Highpass filters: block all frequencies below some *cutoff* frequency.
Example: knock out 60 Hz hum.

(iii) Bandpass filters: block all frequencies outside of $[\hat{\omega}_1, \hat{\omega}_2]$.
Example: radio receiver

(c) Practical filters cannot do any of these exactly.

We will gradually develop methods for better and better filtering: Chaps 5, 6, 7, and 8.

Chap 5	Time-domain analysis	FIR filters Impulse responses Superposition of responses
Chap 6	Frequency-domain analysis	FIR filters sinusoid/exponential responses Superposition of responses
Chap 7	Z-transform	power technique needed for design of filters
Chap 8	IIR filters	time and frequency domain analysis design in z-domain

2.2 Spectral Shaping

Enhance/reduce spectral components digitally.
Examples include equalizers (bass/treble control).

2.3 Signal Detection

Filter as a running correlator Correlation:

$$C[x, y] = \sum x[k]y^*[k].$$

(a) Filters perform running/sliding/moving correlation.

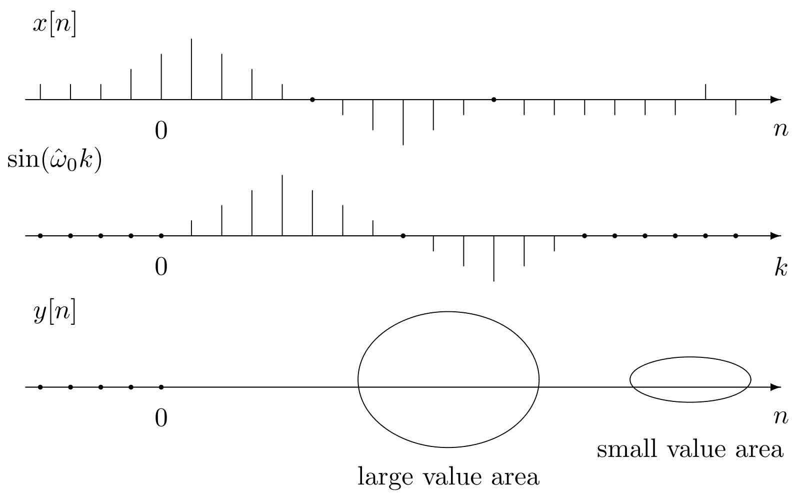
(b) This is useful for signal detection.

(c) (Example) $y[n] = \frac{1}{4}x[n-2] + \frac{1}{4}x[n-1] + \frac{1}{2}x[n]$.
$y[n]$ is the running correlation of $x[n]$ with

$$c[n] = (\ldots, 0, \frac{1}{4}, \frac{1}{4}, \frac{1}{2}, 0, \ldots)$$

(d) Example What does the following do?

$$y[n] = \sum_{k=0}^{50} \sin(\hat{\omega}_0 k) x[n-k].$$



(i) This does running correlation with $\sin(\hat{\omega}_0 k)$.

(ii) It produces large output values when the input is similar to sinusoid with frequency $\hat{\omega}_0$

(iii) It produces small output values when the input is similar to sinusoid with frequency far from $\hat{\omega}_0$

(iv) So it picks up signals with frequency around $\hat{\omega}_0$

(v) It is a bandpass filter or a detector of signal $\sin(\hat{\omega}_0 k)$.