Problems

Elementary Signal Characteristics

1. State the defining formula for:

   (a) The support interval of a continuous-time signal signal $x(t)$.

   (b) The duration of a continuous-time signal $x(t)$.

   (c) The mean value of the continuous-time signal $x(t)$ over the time interval $[t_1,t_2]$.

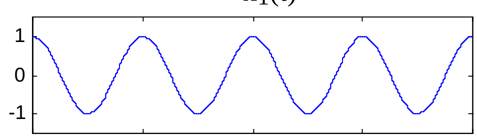
   (d) The average power of the continuous-time signal $x(t)$ over the time interval $[t_1,t_2]$.

   (e) The energy of the continuous-time signal $x(t)$ over the time interval $[t_1,t_2]$.

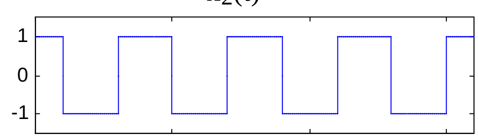
2. The continuous-time signal $x(t) = 3\cos(2\pi 100t)$ is sampled with sampling interval $T_s = 0.005$ msec, creating the discrete-time signal $x[n]$. Find a simple formula for $x[n]$ that does not include a cosine or any other trigonometric function.

3. Consider the two continuous-time signals shown below:

$x_1(t)$

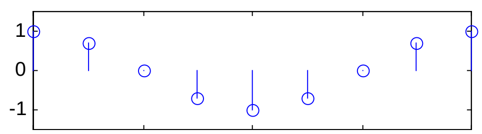


$x_2(t)$

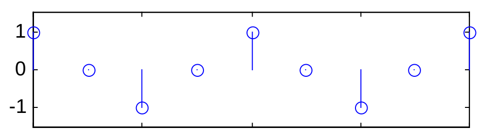


From which of the above signals could each of the following discrete-time signals be obtained by sampling. Find the sampling interval in each case. If more than one sampling interval is possible, find the smallest among those that are possible.

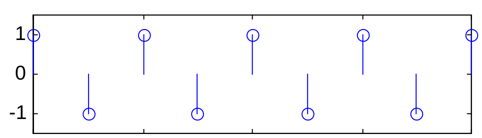
(a) $y_1[n]$



(b) $y_2[n]$

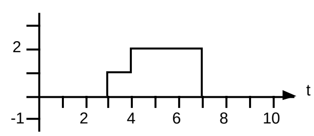


(c) $y_3[n]$

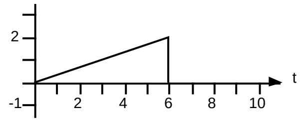


4. Find the support interval, the mean value, the mean-squared value, and the energy of each the following signals, with the last three values computed over the support interval of the signal.

(a) $x_1(t)$



(b) $x_2(t)$



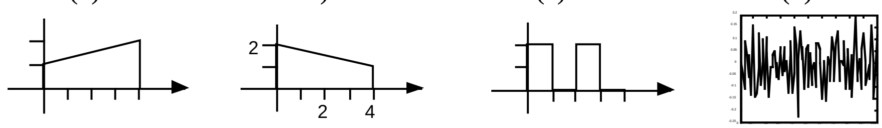
5. Derive the relationship between the mean-squared value, the variance and the mean value:

$$MS(x) = \sigma^2(x) + M^2(x)\ (x)$$

6. Match each signal below with its signal value distribution.

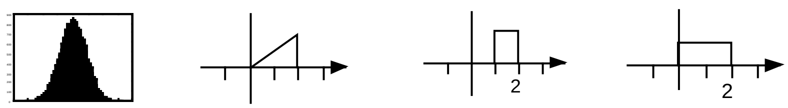
Signals:

(a) b) (c) (d)

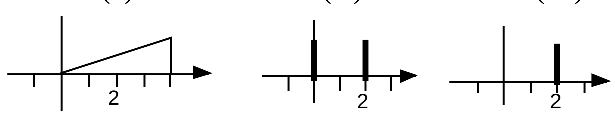


Signal value distributions:

(i) (ii) (iii) (iv)

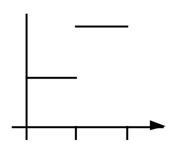


(v) (vi) (vii)

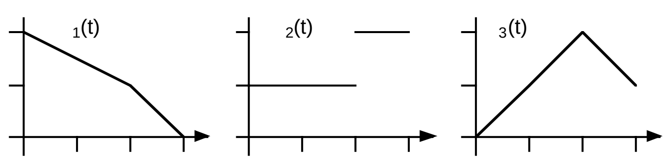


(Tip: You can work this problem from both ends. For each signal, you can look at the range of signal values, and see what you can deduce about which values occur more frequently than others. Also, look at each signal value distribution and see what you can deduce about the signal from which it came.)

7. The function below

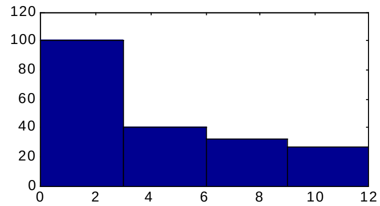


is the signal value distribution of which of the following signals



. . .

8. A continuous-time signal has the histogram shown below.



(a) Find, approximately, the mean value of $x[n]$.

(b) Find, approximately, the mean-squared value of $x[n]$.

Periodicity

9. (a) State the condition defining the periodicity of a signal $x(t)$.

(b) State the definition of the fundamental period of a periodic signal $x(t)$.

10. Which of the signals shown below are periodic? For those that are periodic, find their fundamental period.

(a) $x_a(t) = 3\sin(2t)$

(b) $x_b(t) = 4\sin(e^t)$

(c) $x_c(t) = \cos(t^2+2t+1)$

(d) $x_4(t) = 4(-1)^{\text{floor}(t/3)}$, where floor(z) = largest integer $\leq z$

11. Let $s(t) = A\cos(\omega t+\phi)$.

(a) Show that $s(t)$ is periodic with fundamental period of $2\pi/\omega$.

(b) Find the mean value of $s(t)$ over one period.

(c) Show that the average power of this signal over one period is $A^2/2$.

12. Show that if $x(t)$ and $y(t)$ are periodic with period T, and a and b are arbitrary numbers, then $z(t) = a\,x(t) + b\,y(t)$ is also periodic with period T.

13. (a) Show that if $x(t)$ is periodic with period T and a is a positive number, then $y(t) = x(at)$ is periodic with period T/a.

(b) Repeat Part (a) with the word "period" replaced by "fundamental period".

14. Which of the signals below are periodic? For those that are periodic, find their fundamental period.

(a) $x_a(t) = \cos(2t) + \sin(3t)$

(b) $x_b(t) = \cos(2\pi t) + \sin(6\pi t)$

(c) $x_c(t) = \cos(2t) + \sin(6\pi t)$

15. Let $x(t) = 3\cos(2t)$. Is $y(t) = 4\,x(2t-3)$ periodic? If so, find its fundamental period.

Envelope

16. Find a formula for the envelope of the signal $x(t) = \sin(100t)\sin(3t)$.

Elementary Operations on One Signal

17. Let $y(t) = x(t) + c$. Let (t_1,t_2) be the time interval of interest.

(a) Derive a formula for the mean M(y) of y(t) in terms of c and the mean M(x) of x. (Hint: Start by writing the defining formula for what you need to find, namely, for M(y).)

(b) Derive a formula for the mean-squared value MS(y) of y(t) in terms of c, the mean-squared value MS(x) of x, and the mean value M(x). (Hint: Start by writing the defining formula for what you need to find, namely, for MS(y).)

18. Let y(t) = c x(t) . Let (t_1,t_2) be the time interval of interest.

(a) Derive a formula for the average M(y) of y(t) in terms of c and the mean M(x) of x.

(b) Derive a formula for the mean-squared value MS(y) of y(t) in terms of c, the mean-squared value MS(x) of x, and the mean value M(x).

19. Let y(t) = a x(t) + b. Let (t_1,t_2) be the time interval of interest.

(a) Derive a formula for the mean M(y) of y(t) in terms of a, b, and the mean M(x) of x.

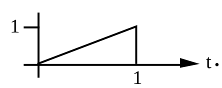
(b) Derive a formula for the mean-squared value MS(y) of y(t) in terms of a, b, the mean-squared value MS(x) of x, and the mean value M(x).

20. Let y(t) = x(at), where x(t) is a signal with support interval (t_1,t_2).

(a) Find the support interval of y(t).

(a) Derive a formula for the mean M(y) of y(t), over its support interval, in terms of a and the mean M(x) of x.

(b) Derive a formula for the mean-squared value MS(y) of y(t), over its support interval, in terms of a and the mean-squared value MS(x) of x.

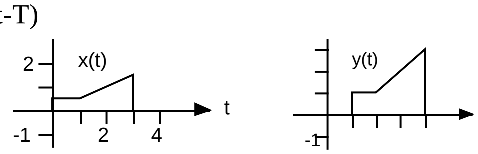
21. Let x(t) = 

. Plot the following signals:

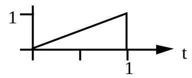
(a) $y_1(t) = -2\, x(3t-2)$

(b) $y_2(t) = 3\, x(-2t+6)$

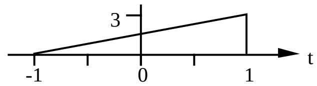
22. Let x(t) and y(t) be as shown below. Find numbers a and T such that y(t) = a x(t-T)



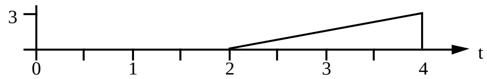
(No systematic procedure has been developed to solve this problem. Use your creativity.)

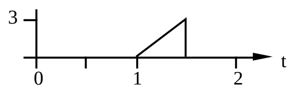
23. Let $x(t) =$ 

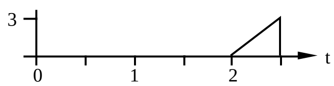
Which of the following shows $y(t) = 3x\left(\frac{t}{2} - 1\right)$?

(a) 

(b) 

(c) 

(d) 

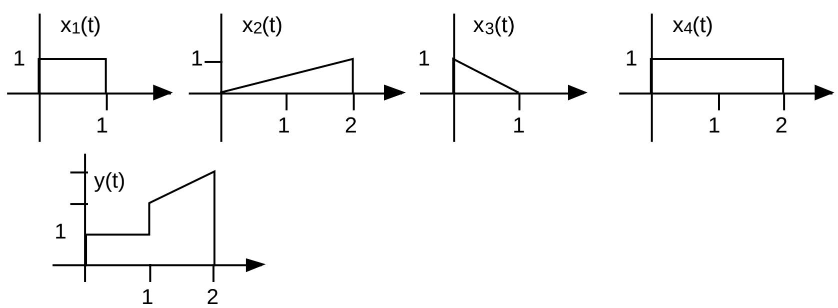
(e) 

Elementary Operations on Two or More Signals

24. For the signals shown below, find numbers a_1, a_2, a_3, a_4 such that

$$y(t) = a_1\, x_1(t) + a_2\, x_2(t) + a_3\, x_3(t) + a_4\, x_4(t)\ .$$

(The a_i's can be positive, negative or zero.)



(Hint: You might think about choosing the a_i's to match some particular feature of $y(t)$ first.)

25. For each of the following, either prove the statement or provide a simple example demonstrating that the statement is false.

(a) The duration of $z(t) = x(t) + y(t)$ equals the sum of the durations of $x(t)$ and $y(t)$.

(b) The energy of $z(t) = x(t) + y(t)$ equals the sum of the energies of $x(t)$ and $y(t)$.

(c) The mean of $z(t) = x(t) + y(t)$ equals the sum of the means of $x(t)$ and $y(t)$.

26. For each of the following collections of signals, determine whether or not the sum is periodic. Find the fundamental period of the periodic ones.

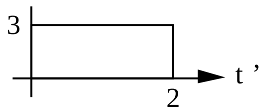
(a) $x(t) = 3\sin(20\pi t)$, $y(t) = 4\cos(40\pi t)$

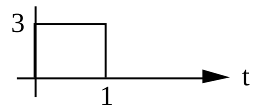
(b) $x(t) = 3\sin(20\pi t)$, $y(t) = 4\cos(21\pi t)$

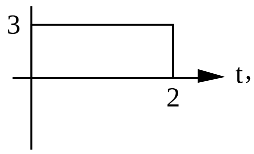
(c) $x(t) = 3\sin(2t)$, $y(t) = 4\cos(\sqrt{2}t)$

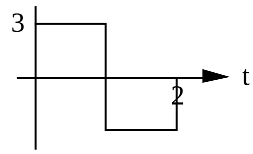
(d) $x(t) = 3\sin(20\pi t)$, $y(t) = 4\cos(40\pi t)$, $z(t) = 3\sin(50\pi t)$

27. For each of the following pairs of signals, find E(x), E(y) and E(x+y) and compare E(x) + E(y) to E(x+y).

(a) x(t) = 

, y(t) = 

(b) x(t) = 

, y(t) = 

28. Let $z(t) = x(t) + y(t)$. Show that $E(z) = E(x) + 2\,C(x,y) + E(y)$.

Mean-Squared Error

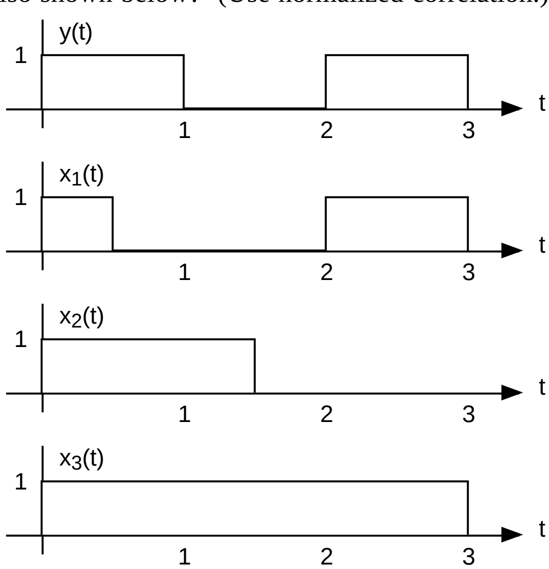
29. Assuming x(t) and y(t) are known signals, find an expression for the value of α that $MSE(x,\alpha y)$ is minimized.

Signal Correlation

30. (a) State the defining formula for the correlation between two continuous-time signals s(t) and r(t), where the time interval of interest is (3,4).

(b) State the defining formula for the correlation between two continuous-time signals s(t) and r(t), where the time interval of interest is (3,4).

31. Which of the signals $x_1(t)$, $x_2(t)$, $x_3(t)$ shown below is most correlated with y(t), also shown below? (Use normalized correlation.)



32. Show that normalized correlation $C_N(x,y)$ between signals $x(t)$ and $y(t)$ equals the unnormalized correlation $C(x',y')$ between the normalized versions of these signals $x'(t) = x(t)/\sqrt{E(x)}$ and $y'(t) = x(t)/\sqrt{E(y)}$, which have energy 1.

33. Show that normalized correlation is not affected by a scaling of either signal, i.e. $C_N(ax,y) = C_N(x,y)$.

34. Show that correlation is linear, i.e. $C(a\,x_1+b\,x_2,y) = a\,C(x_1,y) + b\,C(x_2,y)$.

35. Show that if $x(t)$ and $y(t)$ are uncorrelated, then energy of their sum equals the sum of their energies, i.e. $E(x+y) = E(x+y)$.

36. Show that a constant signal $x(t) = a$ is uncorrelated with any signal $y(t)$ that has zero mean, i.e. if $M(y) = 0$, then $C(x,y) = 0$.

37. Show that normalized correlation between $x(t) = \cos(\omega t)$ and $y(t) = \cos(\omega t+\phi)$ is $C_N(x,y) = \cos(\phi)$.

38. Suppose $x(t) = 2t,\ 0\le t\le 1$ and $x(t) = 0$, for other t, and suppose $y(t)$ is a signal with energy 3 such that $C_N(x,y) = -1$. Find $y(t)$.

39. Continuous-time signals $x(t)$ and $y(t)$ each have support interval $[0,3]$ and average value 2. Their (unnormalized) correlation is $C(x,y) = 1$. If $z = 3y+2$, find $C(x,z)$.

40. Continuous-time signals $x(t)$ and $y(t)$ have $E(x) = 1$, $E(y) = 2$, and $E(x+y) = 5$, where $E(\ )$ denotes energy. Find the (unnormalized) correlation $C(x,y)$.

41. Continuous-time signals $x(t)$ and $y(t)$ have energies $E(x) = 2$, $E(y) = 2$, and are uncorrelated. Find $E(x-y)$.

42. Let $z(t) = x(t) + y(t)$. Let (t_1,t_2) be a time interval of interest. Show that

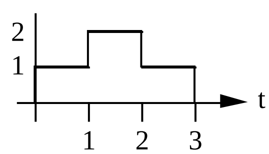
$$E(z) = E(x) + 2\,C(x,y) + E(y),$$

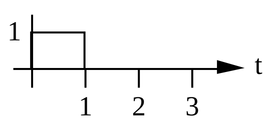
where as usual E denotes energy and C denotes correlation.
(One may conclude from this that $E(z) = E(x) + E(y)$ when and only when x and y are uncorrelated.)

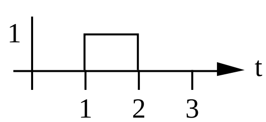
43. given $E(x)$, $E(y)$ and find $E(x+y)$ find $C_N(x,y)$.

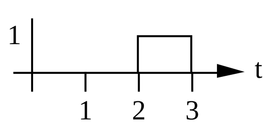
44. given $E(x)$, $C(x,y)$, what can be said about $E(y)$?

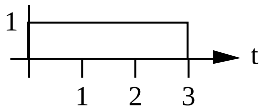
Sgnal components

45. Find the component of the signal $x(t) =$  that is like:

(a) $p_1(t) =$ 

(a) $p_2(t) =$ 

(a) $p_3(t) =$ 

(a) $p_4(t) =$ 

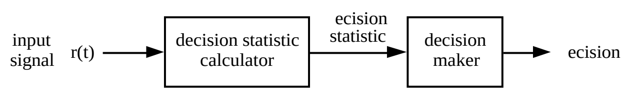
Basic Signal Processing Tasks

46. Consider the running average filter such that when the input signal is $x(t)$, the output signal is

$$y(t) = \frac{1}{2} \int_{t-2}^{t} x(s)\ ds$$

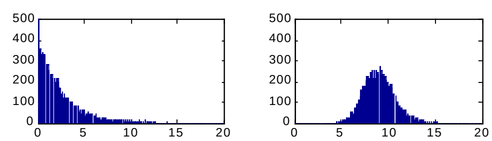
Find an expression for $y(t)$ when $x(t) = \sin(3t)$. Use trigonometric identities to simplify as much as possible.

47. Consider the signal/no signal detection system shown below:



Two histograms for the decision statistic are shown below. Each histogram is based on 10,000 decision statistic measurements.

"no signal present" histogram "signal present" histogram



Assuming we want to minimize the number of decision errors, when the decision statistic is 5, what decision should the decision maker make.

48. A standard audio CD is produced by uniform scalar quantizing the samples of an audio signal and producing 16 bits per sample. The sampling rate is 42,000 samples sec. The quantizer produces 16 bits per sample.

(a) How many quanitzation bins does the quantizer have?

(b) How many bits per second are produced by the quantizer? (Consider just the bits representing the left channel.