Name: $\qquad$
ID Number: $\qquad$
Lab section: $\qquad$
Lecture section: $\qquad$
I have neither given nor received aid on this examination, nor have I concealed any violation of the Honor Code. Signature: $\qquad$

## EECS 206 Exam 2, 2003-3-17

DO NOT TURN THIS PAGE OVER UNTIL TOLD TO BEGIN!

- This is a 120 minute exam.
- It is closed book, closed notes, closed computer.
- You may use two $8.5 \times 11$ " piece of papers, both sides, and a calculator.
- There are 10 problems for a total of 120 points. The questions are not necessarily in order of increasing difficulty.
- This exam has 6 pages. Make sure your copy is complete.
- Continuing to write anything after the ending time is announced will be considered an honor code violation. Fill out your name etc. above now, and do not wait until the end to circle your answers!
- Clearly circle the letter(s) for your answers to problems 3-10 in this table. Any letter(s) that you circle on subsequent pages do not count. We recommend that you show your work on subsequent pages just in case we need to see it.
There are no intentional "none of the above" answers on this exam, but there is always the slim possibility of a typographical error. If you are confident that the correct answer is "none of the above" in any problem, then make a clear mark in this table and show your work clearly near that problem.
Some problems may have multiple possible answers (as indicated), and some partial credit may be awarded for some of these problems. (Each incorrect answer "cancels" a correct answer in such problems.) For problems with only a single answer, no partial credit will be given.

| $1 . \mid$ | (20 points) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2. | $(20$ points) |  |  |  |  |  |
| 3. | a | b | c | d | e | f |
| 4. | a | b | c | d | e | f |
| 5. | a | b | c | d | e | f |
| 6. | a | b | c | d | e | f |
| 7. | a | b | c | d | e | f |
| 8. | a | b | c | d | e | f |
| $\mathbf{9 .}$ | a | b | c | d | e | f |
| 10. | a | b | c | d | e | f |

1. (20 points)

A continuous-time signal $x(t)$ has the following spectrum.


Carefully sketch the spectrum of the following signal: $\quad y(t)=x(t-3 / 4)+x(t+3 / 4)$.
2. (20 points)

Carefully sketch the one-sided spectrum of the following discrete-time periodic signal.

3. (10 points)

A signal $x(t)$ has the following spectrum.


If $x(t)$ is periodic, determine its fundamental frequency $f_{0}$.
a) $x(t)$ is aperiodic
b) $f_{0}=1 / 6$
c) $f_{0}=1 / 3$
d) $f_{0}=1$
e) $f_{0}=3$
f) $f_{0}=6$
4. (10 points)

Consider the following discrete-time signal: $z[n]=3 \cos \left(\pi n+\frac{\pi}{3}\right)+\sin \left(\frac{\pi}{3} n+\frac{\pi}{7}\right)+7 \cos \left(\frac{9 \pi}{4} n+\frac{\pi}{5}\right)$. If $z[n]$ is periodic, determine its fundamental period $N_{0}$.
a) $z[n]$ is aperiodic
b) $8 / 9$
c) 3
d) 4
e) 12
f) 24
5. (10 points)

A discrete-time signal $y[n]$ has the following spectrum.


If $y[n]$ is periodic, determine its fundamental period $N_{0}$.
a) $y[n]$ is aperiodic
b) $N_{0}=2$
c) $N_{0}=3$
d) $N_{0}=5$
e) $N_{0}=6$
f) $N_{0}=10$
6. (10 points)

A 6-periodic signal $x[n]$ has 6-point DFT given by $X[k]=1+(-1)^{k}$. Determine $x[0]+x[3]$. (Choose the closest numerical value.)
a) 10
b) 11
c) 12
d) 13
e) 14
f) 15
7. (10 points)

Unfortunately, the modulator unit is becoming defective and it is producing a slightly contaminated modulation signal as follows: $m(t)=\cos \left(2 \pi f_{c} t\right)+0.01 \cos \left(2 \pi f_{d} t\right)$, with $f_{c}=600 \mathrm{kHz}$ and $f_{d}=606 \mathrm{kHz}$.
Assuming that $x(t)$ is a sinusoidal signal with frequency 4 kHz , which of the following sinusoidal frequency components are present in the transmitted signal $y(t)$ ?
Circle all that apply in the answer table.
a) 600 kHz
b) 602 kHz
c) 604 kHz
d) 606 kHz
e) 608 kHz
f) 610 kHz
8. (10 points)

A continuous-time periodic signal $x(t)$, having period 10 milliseconds, has Fourier series coefficients given by

$$
\alpha_{k}= \begin{cases}\frac{1+|k|}{2}, & -5 \leq k \leq 5 \\ 0, & \text { otherwise } .\end{cases}
$$

An approximation to this signal is synthesized as follows:

$$
\hat{x}(t)=\sum_{k=-3}^{3} \alpha_{k} \mathrm{e}^{\jmath 2 \pi 100 k t}
$$

Quantify the approximation error by determining $\operatorname{RMS}(\hat{x}-x)$. (Choose the closest numerical value).
a) 1.5
b) 2.5
c) 3.5
d) 4.5
e) 5.5
f) 6.5
9. (10 points)

A guitarist plays a 4 kHz sinusoidal signal $x(t)$ with amplitude $A=2$ and phase $\phi=0$ through an amplifier as follows:

$$
x(t) \rightarrow \text { amplifier } \rightarrow y(t) .
$$

An ideal amplifier would produce the output signal $y_{\text {ideal }}(t)=10 x(t)$, but this guitarist has turned up volume so much that the amplifier behaves nonlinearly and produces a distorted output signal of the following form:

$$
y(t)=100 x(t)+10 x^{3}(t) .
$$

Which of the following sinusoidal frequency components are present in the output signal $y(t)$ ?

## Circle all that apply in the answer table.

a) 0 kHz
b) 4 kHz
c) 7 kHz
d) 8 kHz
e) 12 kHz
f) 16 kHz
10. (10 points)

Musicians, especially guitarists, sometimes use an electronic device called an octave doubler to "enhance" the signals produced by their instruments. A typical octave double works by taking the signal and squaring it, and adding the result to the original signal. Specifically, if the input signal is $x(t)$, then the output signal is

$$
y(t)=x(t)+x^{2}(t) .
$$

Suppose the input signal $x(t)=2 \cos \left(2 \pi f_{0} t+\pi / 3\right)$ with $f_{0}=1 \mathrm{kHz}$. Let $\alpha_{k}$ denote the 0.001 -second Fourier series of the output signal $y(t)$. Determine the value of the product $\alpha_{1} \alpha_{2}$.
a) -1
b) 0
c) 1
d) $e^{J \pi / 3}$
e) $\mathrm{e}^{3 \pi / 3}$
f) 2

