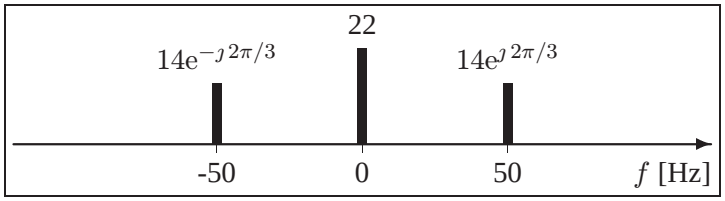
Solutions to EECS 206 Exam 2, 2003-3-17

Regrade requests must be submitted to Prof. Fessler within 1 week of when the exam was returned in your lab section, with a written cover sheet explaining the request clearly. All problems will be re-examined, and scores may increase or decrease.
Discussing the exam with a professor or GSI nullifies the opportunity to submit a regrade request.
No partial credit was given except where indicated below. In particular, "multiple answer" problems that had only one correct answer had to be answered exactly correctly.
(There were multiple versions of the exam so the solutions below may not be in the same order as your exam.)

e2/spectra,shift1

1. (20)
$x(t) = 11 + 14\cos(2\pi 50t - \pi/3)$ so

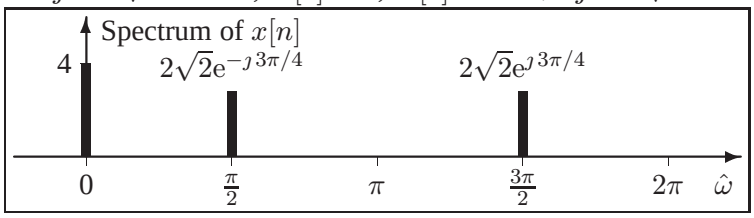
$$\begin{aligned} y(t) &= x(t-3/4) + x(t+3/4) = 11 + 14\cos(2\pi 50(t-3/4) - \pi/3) + 11 + 14\cos(2\pi 50(t+3/4) - \pi/3) \\ &= 22 + 14\cos(2\pi 50t - \pi/3 - 75\pi) + 14\cos(2\pi 50t - \pi/3 + 75\pi) \\ &= 22 + 14\cos(2\pi 50t - \pi/3 + \pi - 2\pi 38) + 14\cos(2\pi 50t - \pi/3 + \pi + 2\pi 37) \\ &= 22 + 28\cos(2\pi 50t + 2\pi/3). \end{aligned}$$



grading: -5 wrong freq., -5 wrong phase (even if "just" a sign error or unsimplified), -5 any wrong amplitude *(HW 5-1)*

e2/dft,spectra1

2. (20)
$X[k] = \frac{1}{4}\left[8e^{-\jmath\frac{2\pi}{4}k} + e^{-\jmath\frac{2\pi}{4}k2}\right] = 2e^{-\jmath\frac{\pi}{2}k} + 2e^{-\jmath\pi k}$,
so $X[0] = 4$, $X[1] = -2 - 2\jmath = 2\sqrt{2}e^{-\jmath 3\pi/4}$, $X[2] = 0$, $X[3] = -2 + 2\jmath = 2\sqrt{2}e^{\jmath 3\pi/4}$,



grading: similar to previous problem. *(HW 7-3)*

e2/spectra,period3

3. (10)
(e) $x(t)$ is periodic with $T_s = LCM(1/6, 1/9, 1/15) = 1/3$, so $f_s = 3$. *(HW 6-3)*

e2/dt,period1

4. (10)
(f) Using the periods of the three components: LCM(2, 6, 8) = 24 *(HW 7-1)*

e2/dt,period2

5. (10)
(a) $y[n]$ is aperiodic since $\hat{\omega} \neq 2\pi M/N$. *(HW 8-2)*

e2/idft1

6. (10)
(c) By the synthesis formula: $x[n] = 2 + 2\mathrm{e}^{\jmath\frac{2\pi}{6}2n} + 2\mathrm{e}^{\jmath\frac{2\pi}{6}4n} = 2 + 4\cos(\frac{2\pi}{6}2n)$. $x[0] = 6$, $x[3] = 12$, so $x[0] + x[3] = 12$. *(HW 8-1)*

e2/am,mod1

7. (10)
(a)(b)(c)(d)(f). Grading: -2 for each omission and incorrect answer.
$y(t) = [\cos(2\pi f_x t + \phi) + 3][\cos(2\pi f_c t) + \cos(2\pi f_d t)]$.
which has frequency components at $f_c, f_c \pm f_x, f_d, f_d \pm f_x$. *(HW 6-1)*

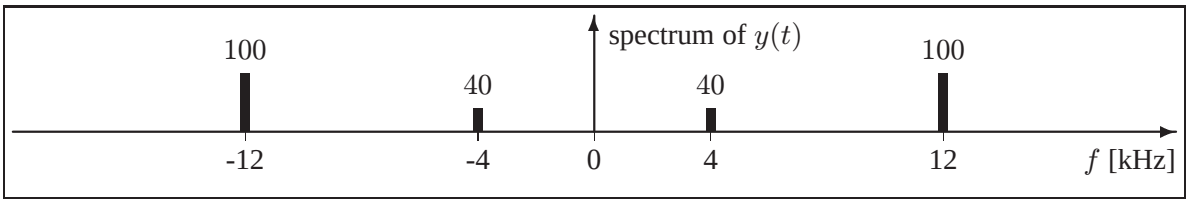
e2/fs,trunc,rms

8. (10)
(e) By Parseval, $\mathrm{MS}(\hat{x} - x) = 2\sum_{k=4}^{5} |\alpha_k|^2 = 2[\left(\frac{1+4}{2}\right)^2 + \left(\frac{1+5}{2}\right)^2] = 2[25/4 + 9] = 61/2 = 30.5$.
So $\mathrm{RMS}(\hat{x} - x) = \sqrt{30.5} \approx 5.5$. *(HW 7-4)*

e2/guitar,nonlin

9. (10)
(b)(e). Grading: -5 for each omission or wrong answer. For $f_0 = 4$kHz we have:

$$\begin{aligned} y(t) &= 100x(t) + 10x^3(t) = 100A\cos(2\pi f_0 t + \phi) + 10[A\cos(2\pi f_0 t + \phi)]^3 \\ &= 200\cos(2\pi f_0 t + \phi) + 80\left[\frac{3}{4}\cos(2\pi f_0 t + \phi) + \frac{1}{4}\cos(2\pi 3 f_0 t + 3\phi)\right] \\ &= 80\cos(2\pi f_0 t) + 200\cos(2\pi 3 f_0 t). \end{aligned}$$

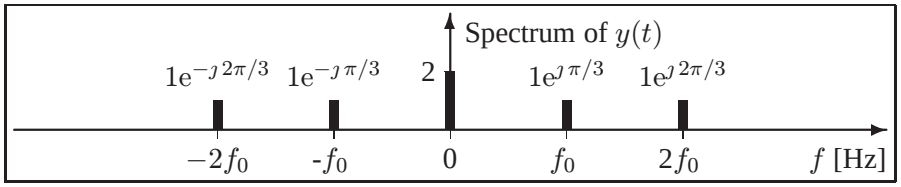


(HW 5-2)

e2/octave,doubler

10. (10)
(a)

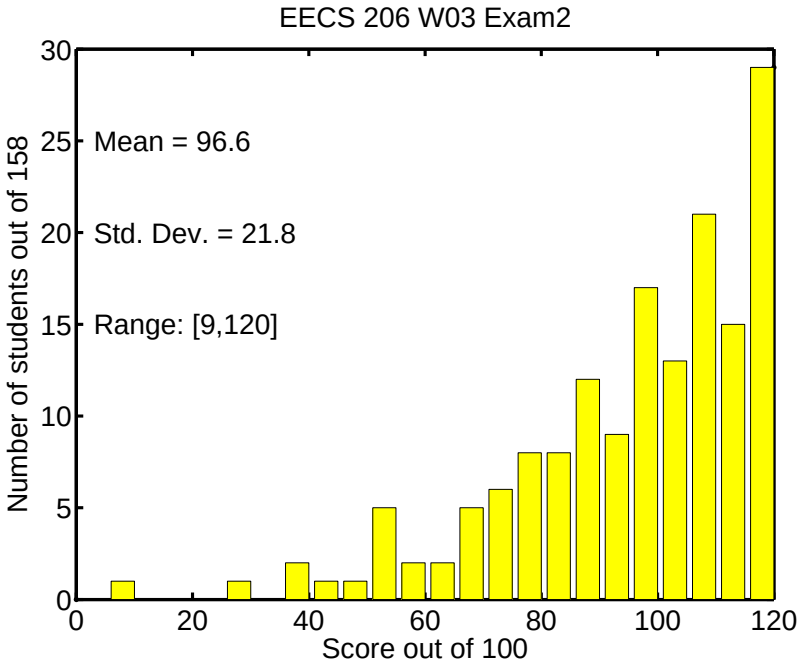
$$\begin{aligned} y(t) &= x(t) + x^2(t) = 2\cos(2\pi f_0 t + \phi) + 4\cos^2(2\pi f_0 t + \phi) \\ &= 2 + 2\cos(2\pi f_0 t + \phi) + 2\cos(2\pi 2 f_0 t + 2\phi), \end{aligned}$$



So by coefficient matching, $\alpha_k = \begin{cases} 2, & k = 0 \\ 1\exp(\pm\jmath\pi/3), & k = \pm 1 \\ 1\exp(\pm\jmath 2\pi/3), & k = \pm 2 \\ 0, & \text{otherwise}, \end{cases}$ so $\alpha_1\alpha_2 = -1$.

(HW 5-2, 6-2)

158 students, mean=96.6/120=80.5%, std=21.8



Rough grades (out of 120):
110+ A
95+ B+ / B
80+ B- / C+
75+ C
70+ C-
69- D's and F's

For elaboration on these solutions, please come to office hours.