Solutions to EECS 206 Exam 3, 2003-4-23

Regrade requests must be submitted to Prof. Fessler *in writing*, by May 2. All problems will be re-examined, and scores may increase or decrease.
Discussing the exam with a professor or GSI nullifies the opportunity to submit a regrade request.
(There were multiple versions of the exam so the solutions below may not be in the same order as your exam.)

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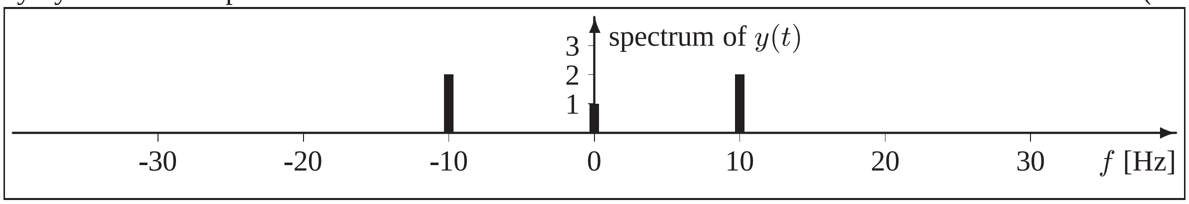
e3/antialias,sinc1

1. (10)
Since $f_s = 1/T_s = 30$, frequencies at and above $f_s/2 = 15$Hz are removed. Everything else is reconstructed perfectly by the sinc interpolator. *(HW 9-4)*



grading: -8 for 20 or 30Hz components. -5 for discrete-time freq.

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e3/nyquist1

2. (10)
Yes. The maximum frequency f_0 of $x(t)$ satisfies $\frac{\pi}{4} = 2\pi \frac{f_0}{f_s}$ so $f_0 = \frac{1}{8} f_s = 125$Hz, so to prevent aliasing we could sample at any rate $\boxed{f_s > 250 \text{ Hz.}}$ *(HW 8-7)*
grading: -3 for ≥ 250Hz

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e3/fir,dft,2

3. (10)
The input signal is $x[n] = 2 + 3\cos(\pi n)$. The frequency response is $\mathcal{H}(\hat{\omega}) = 1 + 2e^{-\jmath 4\hat{\omega}}$, so $\mathcal{H}(0) = 3$, and $\mathcal{H}(\pi) = 3$. Thus $\boxed{y[n] = 6 + 9\cos(\pi n).}$ *(HW 11-10)*

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e3/sin,pz1

4. (10)
$H(z) = 2\frac{(z-0.5-0.5\jmath)(z-0.5+0.5\jmath)}{z(z+1)} = 2\frac{z^2-z+0.5}{z^2+z}$
$\Rightarrow \mathcal{H}(\pi/4) = H\left(e^{\jmath\pi/4}\right) \approx 0.207 + \jmath 0.328 \approx 0.388 e^{\jmath 1.008}$
$\Rightarrow \boxed{y[n] = 3\,|\mathcal{H}(\pi/4)| \cos(\frac{\pi}{4}n + \angle\mathcal{H}(\pi/4)) \approx 1.16\cos(\frac{\pi}{4}n + 1.008).}$ *(HW 13-5,12-5)*
grading: -2 for no gain=2. -4 for wrong phase.

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e3/x,Ho,1

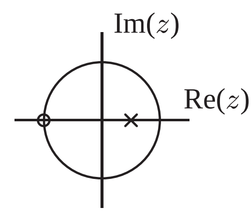
5. (10)
$y[n] = (7\delta[n+1] + 9\delta[n-3]) * h[n] - 6\mathcal{H}(0) = (7\delta[n+1] + 9\delta[n-3]) * (2\delta[n-1] + 3\delta[n-5]) - 6 \cdot 5 =$
$\boxed{14\delta[n] + 39\delta[n-4] + 27\delta[n-8] - 30.}$ *(HW 12-8)*
grading: 5pt for -30, 5pt for δ's. -3 for $Z^{-1}\{14\}$ "=" 14 rather than $Z^{-1}\{14\} = 14\delta[n]$.

e3/Ho,hn,1

6. (10)
$H(z) = \frac{z^{-2}-5z^{-3}}{1+z^{-1}} = z^{-2}\frac{1}{1+z^{-1}} + 5z^{-3}\frac{1}{1+z^{-1}} \Rightarrow \boxed{h[n] = (-1)^{n-2}u[n-2] - 5(-1)^{n-3}u[n-3]\,.}$ *(HW 12-3)*

e3/hn,pass

7. (10)
(d). The system function is $H(z) = \frac{1}{1-\frac{1}{2}z^{-1}} + z^{-1}\frac{1}{1-\frac{1}{2}z^{-1}} = \frac{z+1}{z-\frac{1}{2}}$ and the pole-zero plot is shown to the right. This is a $\boxed{\text{lowpass}}$ filter since the magnitude response at low frequencies is much greater than the response at high frequencies. *(HW 13-5)*



e3/hn,diffeq

8. (10)
The system function is $H(z) = \frac{1}{1-\frac{1}{2}z^{-1}} + z^{-1}\frac{1}{1-\frac{1}{2}z^{-1}} = \frac{1+z^{-1}}{1-\frac{1}{2}z^{-1}} = Y(z)\,/X(z)$ so the difference equation is

$$\boxed{y[n] = \frac{1}{2}y[n-1] + x[n] + x[n-1]\,.}$$ *(HW 13-6)*

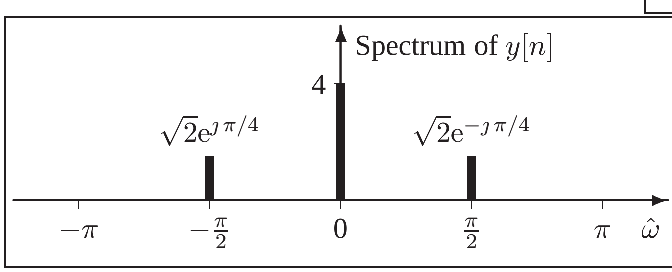
grading: 5pts for $H(z)$, 5pts for diffeq

e3/sample,null1

9. (10)
After sampling, the sinusoid has frequency $\omega_0 = 2\pi f_0/f_s = 2\pi 125/500 = \pi/2$. So we need $\mathcal{H}(\hat{\omega})$, the overall frequency response of the discrete-time filter, to be zero at $\omega_0 = \pi/2$. The overall system function is $H(z) = H_1(z) + H_2(z) = 1 + \frac{1}{2}z^{-3} + bz^{-1} + z^{-2}$, so the frequency response is $\mathcal{H}(\pi/2) = H\big(e^{\jmath\pi/2}\big) = 1 + \frac{1}{2}e^{-\jmath 3\pi/2} + be^{-\jmath\pi/2} + e^{-\jmath\pi} = 1 + \frac{1}{2}\jmath - b\jmath - 1 = (\frac{1}{2} - b)\jmath$ so we need $\boxed{b = 1/2.}$
grading: 0 for $H(z)$. 2 for $\mathcal{H}(\hat{\omega})$. 2 for $\hat{\omega}$. 6 for b.

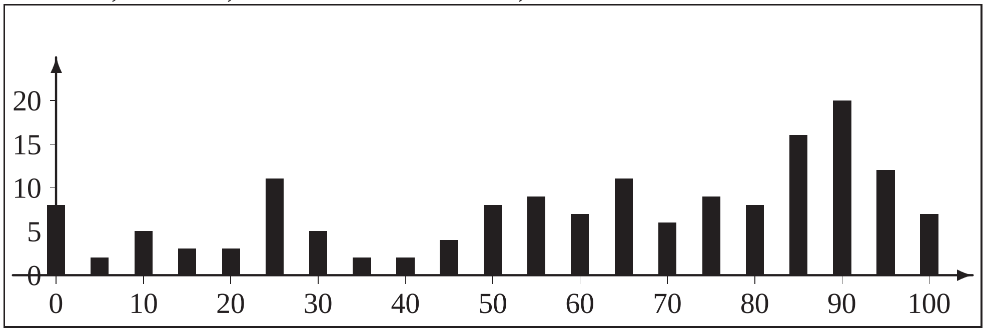
e3/filt2,mult

10. (10)
$\mathcal{H}_1(\hat{\omega}) = 1 - e^{-\jmath\hat{\omega}}$ so $\mathcal{H}_1(\pi) = 2$, $\mathcal{H}_1(\pi/2) = 1 + \jmath = \sqrt{2}e^{\jmath\pi/4}$.
Thus $y_1[n] = 2\cos(\pi n) + \sqrt{2}\cos(\frac{\pi}{2}n + \pi/4)$.
$\mathcal{H}_2(\hat{\omega}) = 1 + e^{-\jmath 2\hat{\omega}}$, so $\mathcal{H}_2(\pi) = 2$, $\mathcal{H}_2(\pi/2) = 0$.
Thus $y_2[n] = 2\cos(\pi n)$.
Multiplying: $y[n] = y_1[n]\,y_2[n] = \big[2\cos(\pi n) + \sqrt{2}\cos(\frac{\pi}{2}n + \pi/4)\big]\,2\cos(\pi n) = \boxed{4 + 2\sqrt{2}\cos(\frac{\pi}{2}n - \pi/4)}$



grading: -3 for $\oplus$ instead of $\otimes$. -3 if $3\pi/2$ and $\pi/2$ not combined. 5pts for $y_1[n]$ and $y_2[n]$. 5pts for spectrum.

158 students, mean=61.3, std=29.7, 4 students scored 100%, 8 students scored 0%



Rough grades (out of 100):
85+ A
75+ B/B+
60+ C+/B-
55+ C
50+ C-
49- D's and below

For elaboration on these solutions, please come to office hours.