This final exam review is meant to be a comprehensive outline to all the topics covered in the course. Examples are not included. Sample problems that demonstrate some of these concepts are given in a separate paper.

- Signal Properties
  - Max, Min, Mean, Energy, Mean Square, RMS
  - Be able to use MSD (just mean square value of the difference) to compare difference (or error) in signals
- Translation and scaling of a signal
  - How signal properties are affected by scaling and shifting in amplitude
  - How signal properties are affected by scaling and shifting in time
  - Determine a the results of shifting/scaling in amplitude and time (i.e., if given $x(t)$, be able to find $y(t) = a + bx(ct + d)$)
    - Recall that shifting in amplitude affects time *outside* of the original support region (e.g. See HW2, problem 4, sketch of $y(t)$)
- Signal Value Distribution (SVD)
  - Essentially a histogram
  - Information on time sequence of the signal is lost (so shifting/scaling of *time* will have no effect on SVD)
  - The *vertical* axis of the signal becomes the *horizontal* axis of the SVD (see HW2, problem 2)
- Periodic signals—sinusoids (estimation of amplitude, frequency, phase)
  - Relate parameters amplitude, frequency, phase to a figure of the signal.
  - Convert to standard form (e.g., use $\sin(t) = \cos(t - \pi/2)$ and keep $-\pi < \Phi \leq \pi$
- Periodic signals—complex exponentials
  - Distinguish between the periodic part versus the phasor
  - Be able to convert to cosine using just the real part
- Complex arithmetic
  - Euler and inverse
  - Have some specific angles memorized (like $e^{j\pi/2} = j$, $e^{j\pi} = -1$, $e^{-j\pi/2} = e^{j3\pi/2} = -j$)
  - Convert between polar (magnitude and phase) and Cartesian form of complex number
  - Know things like $|z|^2 = zz^*$ and similar for division
  - Note that *phasors* are just complex numbers and should be added as such (see section on adding sinusoids)
- Operations on two signals—adding sinusoids of same frequency
  - Convert sinusoids to real part of complex exponential
  - Separate the periodic part from the phasor
  - Deal with phasors as complex numbers
- Adding periodic signals - periodicity of the result
  - If ratio of periods of each periodic signal is rational number⇒periodic
  - If ratio of periods of each periodic signal is irrational number⇒not periodic
- Correlation and normalized correlation is analogous to vector dot product (it indicates how much a signal is in the "direction" of another signal)
- Spectral Representation of Continuous Time Signals (infinite duration)
  - Indicates that the signal is the sum of sinusoids

- Magnitude is symmetric about origin while phase is conjugate symmetric
- Periodicity of the sum depends upon same rules as adding periodic signals (above)
- Spectra and Time Scaling/Shifting
  - If given spectrum of $x(t)$ and $y(t) = a + bx(ct + d)$, be able to calculate spectrum of $y(t)$ (page 3a.10) (Linearity properties).
  - Be able to add/subtract spectra of two signals (addition of two spectral lines at same frequency is just phasor addition)
- Fourier Series
  - A specific type of spectra with frequency components represented as a fundamental and multiples of it
  - Have the synthesis and analysis formulas handy (pp. 3a.14, 3a.15)
  - Should be able to calculate frequency from k-value ($f_k = f_0 \cdot k$ or $f_k = k/T_0$)
  - Parseval's theorem
  - Know (or have written) most of properties (linearity, time shifting, frequency shifting, etc.)

- Spectral Representation of Discrete Time Signals (infinite duration)
  - Discrete-time signal may not be periodic even though it has a single spectral line
    - Frequency ($\hat{\omega}$ in radian/sample) must be rational number times π in order to be periodic
  - If each spectral line in discrete-time spectrum represents periodic signal, then the sum is periodic
  - Be able to convert easily between 1-sided and 2-sided spectrum
  - DFT - analogous to Fourier Series
    - know the analysis and synthesis equations, Parseval's theorem
    - Know (or have written) most of properties (linearity, time shifting, frequency shifting, etc.)
      - Be able to relate k to $\hat{\omega}$ using $\hat{\omega} = 2\pi k/N$
      - Recognize $-k$ is equivalent to $N - k$
      - Combine equivalent frequencies ($\hat{\omega}$ and $\hat{\omega} + 2\pi n$ are equivalent when $n \in \mathbb{Z}$)
    - Important to note–it is NOT infinite
    - Be able to use coefficient matching to quickly do synthesis/analysis

- Sampling
  - Understand $x[n] = x(nT_s)$
  - Convert to/from sampled frequency using $\hat{\omega} = \omega/f_s$
  - Be able to find aliased frequencies (frequencies in the discrete world live on a circle)
    - Case 1 aliasing - aliased but not folded (phase matches that of continuous time signal)
    - Case 2 aliasing - aliased and folded (phase has opposite sign as that of continuous time signal)
  - Given a quantization range and number of quantization levels, determine what the levels are (HW 13, problem 7 and Lab 5)
  - Quantization error and how it affects spectrum (HW 13, problem 7)
  - Sampling <u>at</u> the critical frequency ($\hat{\omega} = \pi$ exactly) $\Leftrightarrow$ aliasing occurs (since we can cannot determine both phase and magnitude)
  - Anti-aliasing filter (a continuous-time filter that acts <u>before</u> sampling occurs) will remove all signals at or above sampled frequency of π (note: cannot realize a perfect anti-aliasing filter!)

- Reconstruction of Sampled Signals
  - Zero-order hold (a pulse drawn at the sample)
  - First-order (linear) interpolator (connect the dots)
  - Ideal (sinc) interpolator (uses all past and present data and can theoretically reconstruct original if not aliased)

- If ideal interpolator assumed, be able to convert between sampled signal and corresponding continuous time signal by inspection

- Linear, time-invariant systems (LTI)
  - Linear defined by possessing properties of superposition and homogeneity
    - Superposition - adding two input signals is equivalent to inputting each separately then adding the outputs
    - Homogeneity - scaling an input signal is equivalent to inputting the signal, then scaling the output
  - Time-invariance requires that shifting an input signal be equivalent to shifting the output
  - LTI properties can be used to calculate output in response to an input by characterizing response to scaled, shifted, or superposed versions of it (see HW 10, problem 9b)
- Causal systems
  - Require that output be a function of input at only current and past time
  - Not affected by signals unrelated to input (e.g., constant output or sinusoid output that is not due to input)
- Filtering using Finite Impulse Response (FIR)
  - LTI system
  - Can summarize all we need to know in terms of $h[n]$, impulse response
  - Can represent as difference equation of the form $y[n] = b_0x[n] + b_1x[n-1] + b_2x[n-2] + b_3x[n-3] + \cdots$
  - Impulse response is (almost trivially) obtained by substituting $\delta[n]$ for $x[n]$
  - Convolution used to calculate time domain response to finite-length input signals
  - Simple algorithm (like 3rd grade multiplication) works for this case (p. 132 of textbook) using coefficients of $h[n]$ and of $x[n]$
- Filtering using Infinite Impulse Response (IIR)
  - LTI system (like FIR)
  - Can summarize all we need to know in terms of $h[n]$, impulse response (like FIR)
  - Can represent as difference equation of the form $y[n] = b_0x[n] + b_1x[n-1] + b_2x[n-2] + b_3x[n-3] + \cdots + a_1y[n-1] + a_2y[n-2] + a_3y[n-3] + \cdots$
  - Impulse response is infinite so contains terms like $a^n u[n]$
  - Convolution still applies, but is impossible to work out to completion
- z-transform
  - Simplifies calculations by allowing FIR/IIR filters to be expressed as ratio of polynomials
  - Frequency response is intimately related to location of roots of polynomials
    - Roots of numerator (FIR part) are zeros of system
    - Roots of denominator (IIR part) are poles of system
    - Roots of polynomials are found by factoring, just as in high school algebra class
  - Allows us to express convolution as multiplication of polynomials (which is all it is anyway ...compare page 132 to page 216 of textbook)
  - Calculation of z-transform from difference equation (almost trivial)
    - $y[n] = b_0x[n] + b_1x[n-1] + b_2x[n-2] + \cdots + a_1y[n-1] + a_2y[n-2] + \cdots$
    - $Y(z) = b_0X(z) + b_1X(z)z^{-1} + b_2X(z)z^{-2} + \cdots + a_1Y(z)z^{-1} + a_2Y(z)z^{-2} + \cdots$
  - Cascade of systems is product of z-transforms (product of ratios of polynomials)
  - Be able to use partial fraction expansion (see HW 13)

  - NOTE: WE DO NOT YET KNOW HOW TO TAKE z-TRANSFORM OF INFINITE SUPPORT SIGNALS, only of systems and finite support signals
    * The text computes transfer function in z-domain by ASSUMING the z-transforms of the input and output signals exist
    * Take EECS 306 to learn more about z-transforms of infinite support signals

- Frequency Response
  - Just a complex number with the magnitude and phase functions of $\hat{\omega}$
  - Can simplify only for "simple filters"
    * When symmetric, can pull out the middle term (Example 6.1 of textbook)
    * In this case, phase is linear with frequency (times half the order of the filter)
  - Magnitude is equivalent to the "height" of $H(z)$ (system function) evaluated on the unit circle (good visualization is Figure 8.23 of textbook)
  - Equivalently, $\mathcal{H}(\hat{\omega}) = H(z)|_{z=e^{j\hat{\omega}}}$
  - Magnitude is always a nonnegative number
  - Phase changes in jumps when
    * Phase must change by π to keep magnitude positive
    * Phase must remain between $-\pi$ and π (reducing θ modulo 2π)
  - Might be easier to calculate at a few frequencies if all that is needed (rather than try to simplify to a general magnitude and phase form)
  - Can visualize frequency response magnitude at a point on the unit circle as product of distances from the point to zeros, divided by product of distances from same point to poles (Figure 8.24 of textbook)

- Special Filters
  - N-point moving average
    * Frequency Response - magnitude is Dirichlet function phase is linear (pp. 180,181 of textbook)
    * z-transform has N-1 zeros on unit circle (p. 228 ff of textbook)
  - Nulling filters (designed to place a null or zero at a particular frequency)
    * Filters with real coefficients will null complex conjugate frequencies
    * The set of frequencies nulled by a cascade of nulling filters is the union of the frequencies nulled by each filter
  - Bandpass filters
    * Analogous to a pair of N-point moving average filters "rotated" in the z-plane from $\hat{\omega} = 0$ to another pair of complex conjugate frequencies
    * z-transform has N-2 zeros on the unit circle and another zero inside the unit circle

- Miscellany
  - Signal "suddenly applied" is just product of infinite support signal with unit step function
    * For FIR, results in transient response up to order of filter in length, steady state thereafter
    * For IIR, transient response decreases exponentially but never goes away (in theory)
  - Should be able to sketch frequency response magnitude from pole/zero plot
  - Should be able to sketch impulse response from pole/zero plot
  - If given input to filter and asked to find output
    * Use $h[n]$ and convolution (or equivalently, $H(z)$) to find output due to those components of input that are of the form
      $c_0\delta[n] + c_1\delta[n-1] + c_2\delta[n-2] \ldots c_N\delta[n-N]$
    * Use frequency response $\mathcal{H}(\hat{\omega})$ to find output (steady state) due to those components of input that are of the form $a\cos(\hat{\omega}n + \phi)$
    * DC signals treated as signals at $\hat{\omega} = 0$
    * If input "suddenly applied," then use steady state after transient is gone, but calculate transient using $h[n]$ and convolution.
    * LTI allows summing outputs calculated by all methods