Homework #3, EECS 206, W03. Due **Fri. Jan. 24**, by 11:30AM

Notes

- Review the HW policies on HW1!
- Correction: although the syllabus correctly lists the times for exam1 and exam2 as 6-8PM, the calendar on the web site incorrectly said the exams would be 6-7:30PM. That calendar has been corrected; please correct your copy.
- Reading: remainder of "Part 1" lecture notes, Sections 2.1 to 2.4 of text.

Skill Problems

1. [20] Concept(s): ***signal similarity measures: mean-squared difference***

The signal $x[n] = 5 \cdot 2^{-n}$ for $n = 0, \ldots, 4$ is needed in a certain application, but the available "economy" DSP hardware can only generate *integer* signal values rather than real numbers. So you must find an integer approximation to $x[n]$.

(a) [0] A crude integer approximation to $x[n]$ would be the signal $x_0[n] = 2$, $n = 0, \ldots, 4$, since 2 is the nearest integer to the mean of $x[n]$. Show that $\mathrm{MSD}(x, x_0) = 2.91$.

(b) [10] As measured by mean-squared difference, which of the following two integer-valued signals would be a better approximation to $x[n]$? Explain.

- $y[n] = \lfloor x[n] \rfloor$, where $\lfloor \cdot \rfloor$ denotes the **floor** function that rounds numbers *down* to the nearest integer.
- $z[n] = \lceil x[n] \rceil$, where $\lceil \cdot \rceil$ denotes the **ceiling** function that rounds numbers *up* to the nearest integer.

Hint: you may look at (and use) MATLAB's `floor` and `ceil` functions if you want.

(c) [10] If you could choose any integer-valued signal to approximate $x[n]$, what would you choose? Determine the MSD between your signal and the desired $x[n]$, and compare it to those you found in the previous part.

This problem is realistic in the sense that all DSP chips have finite precision, so any signal that is implemented in practical hardware will be only an approximation to the desired signal.

2. [20] Concept(s): ***correlation***

(a) [0] Verify that $C(x_1, x_2) = 1/3$ for the following pair of signals:

$$x_1(t) = \begin{cases} \mathrm{e}^{-t}, & t \ge 0 \\ 0, & \text{otherwise,} \end{cases} \quad \text{and } x_2(t) = x_1(2t)\,.$$

(b) [10] Determine the correlation between the following pair of signals.

$$y_1[n] = \begin{cases} (1/2)^n, & n \ge 0 \\ 0, & \text{otherwise,} \end{cases} \quad \text{and } y_2[n] = \begin{cases} (-1/2)^n, & n \ge 0 \\ 0, & \text{otherwise.} \end{cases}$$

Hint: see the "useful formulas" page.

(c) [10] Determine the *normalized correlation* between the following pair of signals.

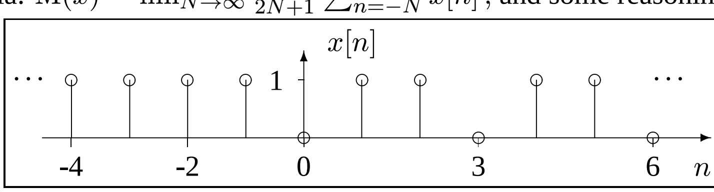
$$z_1(t) = \begin{cases} \mathrm{e}^{-\alpha t}, & t \ge 0 \\ 0, & \text{otherwise,} \end{cases} \quad \text{and } z_2(t) = \begin{cases} \mathrm{e}^{-\beta t}, & t \ge 0 \\ 0, & \text{otherwise.} \end{cases}$$

(d) [0] (Optional.) For a given $\alpha > 0$, what value of $\beta \ge 0$ maximizes the correlation between $y_1(t)$ and $y_2(t)$? What about the normalized correlation? How do these compare with your intuition?

3. [10] Concept(s): ***mean and variance of infinite duration discrete-time signals***

(a) [0] Show that the average value is $\mathrm{M}(x) = 5/6$ for the following discrete-time signal.

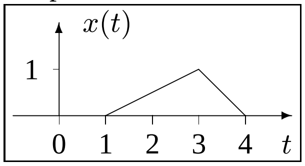
Hint. Use this formula: $\mathrm{M}(x) = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x[n]$, and some reasoning.

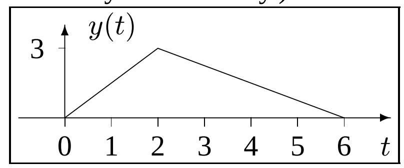


(b) [10] Determine the variance of $x[n]$.

Mastery Problems

4. [10] Concept(s): ***time shift/scale***
Let $x(t)$ and $y(t)$ be the signals shown below. Find numbers a, b, and c such that $y(t) = ax(bt+c)$.
(No systematic process has been developed to solve this problem. Use your creativity.)





5. [10] Concept(s): ***signal similarity measures: MSD***
Let $x[n]$ and $y[n]$ denote two discrete-time signals with the same support interval and duration N.
(a) [0] Show that
$$\mathrm{E}(x-y) = \mathrm{E}(x) + \mathrm{E}(y) - 2C(x,y).$$
(b) [10] Similarly, find a simple relationship between $\mathrm{MSD}(x,y)$ and $\mathrm{MS}(x)$, $\mathrm{MS}(y)$, $C(x,y)$, and N.

6. [10] Concept(s): ***correlation, orthogonality, and harmonic sinusoids***
Let $x_1(t)$ be a sinusoid with frequency f_1 and period $T_1 = 1/f_1$, *i.e.*, $x_1(t) = A_1 \cos(2\pi f_1 t + \phi_1)$, and let $x_2(t)$ be a sinusoid with frequency $f_2 = mf_1$ for any integer $m \neq 1$, *i.e.*, $x_2(t) = A_2 \cos(2\pi f_2 t + \phi_2)$. Such sinusoids are called **harmonically related** and are central to the upcoming topic of Fourier series. Show that $x_1(t)$ and $x_2(t)$ are **orthogonal** over an interval of length T_1 by showing that their correlation is zero: $\int_0^{T_1} x_1(t)\, x_2(t) \,\mathrm{d}t = 0$.
Hint. Use this equality: $\cos(\alpha)\cos(\beta) = \frac{1}{2}\cos(\alpha-\beta) + \frac{1}{2}\cos(\alpha+\beta)$.

Optional Extra Credit Problems

7. [10] Concept(s): ***signal similarity measures: MSD***
Let $x[n]$ be a finite-duration discrete-time signal with support interval $\{n_1, \ldots, n_2\}$.
Let $w[n] = \begin{cases} \alpha, & n = n_1, \ldots, n_2 \\ 0, & \text{otherwise.} \end{cases}$
Find a simple expression for the value of α that minimizes $\mathrm{MSD}(x,w)$.