Homework #10, EECS 206, W03. Due **Fri. Mar. 21**, by 11:30AM

Notes

- Review the HW policies on HW1!
- Continuing to write *anything* on your homework after the 11:30AM due date, or attempting to put your solution into the box any time after 11:30AM is an honor code violation.
- Reading: Text sections 5.1-5.9
- Relevant practice problems on the DSP CDROM: 5.3, 5.12-17, 5.18-29

Skill Problems

1. [15] Concept(s): ***running average system, output from input***
An L-point running average system has the following input-output relationship:

$$y[n] = \frac{1}{L}\sum_{k=0}^{L-1} x[n-k]\,.$$

(a) [5] Sketch the output signal $y[n]$ (over the range $-5 \leq n \leq 10$) when the input signal is given by $x[n] = \delta[n-2] + \delta[n-5]$, for the case $L = 4$.

(b) [5] Sketch the output signal $y[n]$ (over the range $-5 \leq n \leq 10$) when the input signal is the unit-step function $x[n] = u[n]$, for the case $L = 4$.

(c) [5] Determine a general expression for $y[n]$ that will apply for any $L \in \mathbb{N}$ when $x[n] = u[n]$.

2. [10] Concept(s): ***FIR filter, LTI system, difference equation, output from input, impulse response***
An FIR filter is described by the following **difference equation**:

$$y[n] = 2x[n] + x[n-1] - x[n-3]\,.$$

(a) [0] Is this a linear, time-invariant (LTI) system?

(b) [0] The input signal is given by $x[n] = \begin{cases} 0, & n < 0 \\ n+1, & 0 \leq n \leq 3 \\ 2, & n > 3. \end{cases}$ Sketch this signal.

(c) [5] Determine a formula for the output signal $y[n]$. Use braces just like the above formula for $x[n]$.

(d) [0] Sketch the output signal $y[n]$.

(e) [5] Find the **impulse response** $h[n]$ of this system. In other words, determine a output signal $y[n]$ when the input signal is when $x[n] = \delta[n]$. (When $x[n] = \delta[n]$, then $y[n] = h[n]$.)
Give both a formula for $h[n]$ and sketch it.

3. [15] Concept(s): ***discrete-time system properties***

(a) [0] Show that the system described by the input-output relationship $y[n] = (n+1)^3 x[n-2]$ is linear but time varying.

(b) [0] Show that the system described by the input-output relationship $y[n] = x[n]\,x[n-2]$ is nonlinear but time-invariant.

(c) [15] Consider the system with input-output relationship $y[n] = 3x[n-1] + \cos\!\left(\frac{\pi}{2}(n+1)\right)$.
Explain why this system is or is not: linear, time-invariant, and/or causal.

4. [20] Concept(s): ***discrete time systems and their input-output relationships***
A discrete-time system works as follows. Each output signal value is the average of the squares of the four previous input signal values.
   (a) [5] Write down the input-output relationship for this system.
   (b) [5] Is this system causal? Explain.
   (c) [5] Is this system time-invariant? Explain.
   (d) [5] Is this system linear? Explain.

5. [10] Text 5.6. Concept(s): ***FIR coefficients from*** $h[n]$

6. [20] Text 5.7. Concept(s): ***system properties from input-output relationship***

7. [15] Text 5.11. Concept(s): ***cascade of LTI FIR systems***

Mastery Problems

8. [10] Concept(s): ***running average system, geometric series input, step function***
   (a) [0] Sketch the signal $s[n] = (0.5)^n u[n]$.
   Notice the effect of the step function on the signal values for $n < 0$.
   (b) [10] The input signal for a certain L-point running average system is given by $x[n] = a^n u[n]$.
   Determine a general formula for the output signal $y[n]$.
   Hint. Your answer should have braces with 2 or 3 cases.

9. [10] Concept(s): ***linearity and time invariance***
A discrete-time "mystery" system $\mathcal{T}$ is known to be LTI, but its input-output relationship is unknown. It is tested with a couple of input signals, and the following input-output pairs are observed.

$$x_1[n] = \delta[n] - \delta[n-1] \quad \xrightarrow{\mathcal{T}} \quad y_1[n] = \delta[n] - \delta[n-1] + 2\delta[n-3]$$
$$x_2[n] = \cos(\frac{\pi}{3}n) \quad \xrightarrow{\mathcal{T}} \quad y_2[n] = 7\cos(\frac{\pi}{3}n - \frac{\pi}{7})$$

   (a) [0] Sketch the signal $y_1[n] = \delta[n] - \delta[n-1] + 2\delta[n-3]$
   (b) [10] Use linearity and time invariance to determine what that output signal would be if the input signal is given by $x[n] = 7\delta[n] - 7\delta[n-2]$.