Homework #12, EECS 206, W03. Due **Fri. Apr. 4**, by 11:30AM

Notes

- Review the HW policies on HW1!
- Final exam information is on HW11. **We will check UM ID's at the final exam.**
- Reading: Text Ch. 7.
- Relevant practice problems on the DSP CDROM: 7.3, 7.4, 7.18, 7.21, 7.25, 7.32, 7.35, 7.46, 7.49

Skill Problems

1. [0] Text 7.1. Concept(s): ***(z-transform of finite sequences)***

   Hint: $X_4(z) = 2 - 3z^{-1} + 4z^{-3}$

2. [0] Text 7.3. Concept(s): ***(diffeq and response from system function)***

   Hint: $h[n] = \delta[n] + 5\delta[n-1] - 3\delta[n-2] + 2.5\delta[n-3] + 4\delta[n-8]$

3. [30] Text 7.8. Concept(s): ***(diffeq to $h[n]$, $H(z)$, zplane, $\mathcal{H}(\hat{\omega})$)***

4. [15] Text 7.9ace. Concept(s): ***(Cascade of two $H(z)$'s)***

5. [15] Concept(s): ***system function and frequency response***

   A filter has input-output relation

   $$y[n] = x[n-1] + \sqrt{2}x[n-3] + x[n-5]\,.$$

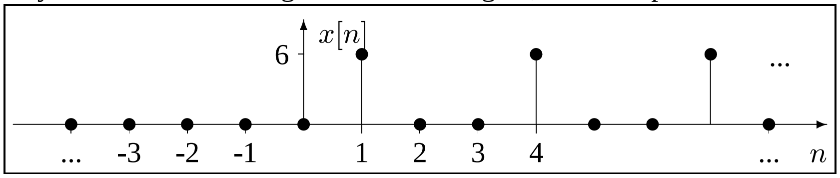
   (a) [5] By taking the Z-transform of both sides of this equation to obtain $Y(z) = H(z)\,X(z)$, identify the system function $H(z)$.

   (b) [5] From the system function of part (a), show that there is a frequency ω_0 for which the output signal is zero when $x[n] = A\cos(\omega_0 n + \phi)$ for any A, ϕ. What is this frequency?

   (c) [5] Determine the output signal $y[n]$ when the input is the above sinusoid with $A = 10$, $\omega_0 = \pi/5$ and $\phi = 0$.

Mastery Problems

6. [25] Concept(s): ***suddenly applied signals***
The input to an LTI system is the following discrete-time signal, which is periodic for $n \geq 0$.



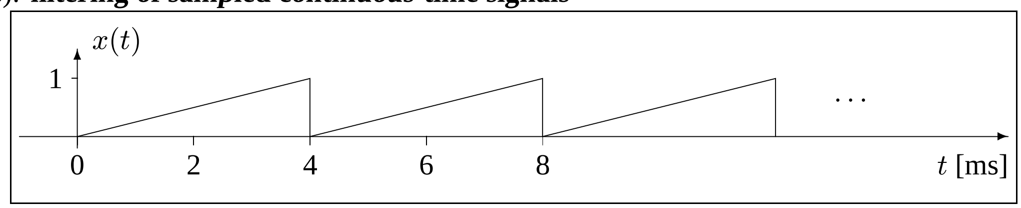
The frequency response of the system is given by

$$\mathcal{H}(\hat{\omega}) = \frac{1}{4}\left(1 + \mathrm{e}^{-\jmath\hat{\omega}} + \mathrm{e}^{-\jmath 2\hat{\omega}} + \mathrm{e}^{-\jmath 3\hat{\omega}}\right).$$

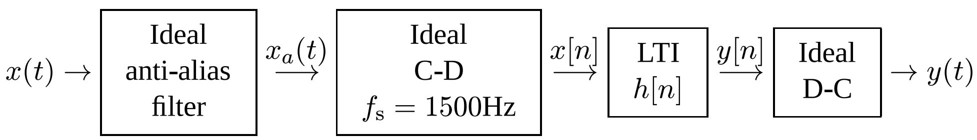
(a) [20] Determine a formula for the output signal $y[n]$ without using braces.

(b) [5] Sketch the output signal $y[n]$, and identify which part is the transient response.

7. [30] Concept(s): ***filtering of sampled continuous-time signals***



The above sawtooth signal $x(t)$ is the input to the following connected systems:



(a) [0] Sketch the spectrum of $x(t)$. (See HW6.)

(b) [5] Sketch the spectrum of $x_a(t)$.

(c) [10] Sketch the spectrum of $x[n]$.

(d) [10] Sketch the spectrum of $y[n]$, assuming that $h[n] = \frac{1}{2}(\delta[n] + \delta[n-1])$.

(e) [5] Determine $y(t)$.

Optional challenge. Design a filter (specify $h[n]$) that would remove all of the harmonics except the fundamental frequency for the above sampled sawtooth signal.

8. [10] Concept(s): ***response to mixed periodic / aperiodic input given*** $H(z)$
Consider the following cascade of LTI systems

$$x[n] \rightarrow \boxed{H_1(z) = 1 + z^{-2}} \rightarrow \boxed{H_2(z) = 1 - z^{-2}} \rightarrow y[n]\,.$$

The input to this cascade system is the signal

$$x[n] = 20 - 7\delta[n-5] + 20\cos(\frac{\pi}{4}n).$$

Determine the output signal $y[n]$. Given an equation (no braces!) valid for all n.
Hint. Do not use convolution. Use more than one method and linearity.

9. [0] Will you need your UM ID at the final exam?