

### Useful Formulas for EECS 206

Geometric series:

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}, \quad \text{if } |a| < 1, \quad \sum_{n=0}^{N-1} a^n = \begin{cases} \frac{1-a^N}{1-a}, & a \neq 1 \\ N, & a = 1 \end{cases}$$

$$\sum_{n=1}^{N-1} na^n = \begin{cases} \frac{a(1-a^N)}{(1-a)^2} - \frac{Na^N}{1-a}, & a \neq 1 \\ \frac{N(N-1)}{2}, & a = 1 \end{cases}$$

$$\frac{1}{N} \sum_{n=0}^{N-1} e^{j\left(\frac{2\pi k}{N}\right)n} = \begin{cases} 1, & k = 0, \pm N, \pm 2N, \dots \\ 0, & \text{otherwise} \end{cases} = \sum_{m=-\infty}^{\infty} \delta[k - mN]$$

Trigonometry:

$$\begin{aligned} \cos(x+y) &= \cos x \cos y - \sin x \sin y \\ \sin(x+y) &= \sin x \cos y + \cos x \sin y \\ \sin \theta &= \cos\left(\theta - \frac{\pi}{2}\right) \\ \cos^2 \theta &= \frac{1}{2}(1 + \cos 2\theta), \quad \sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta) \end{aligned}$$

Euler:

$$e^{j\theta} = \cos \theta + j \sin \theta, \quad \cos \theta = \frac{e^{j\theta} + e^{-j\theta}}{2}, \quad \sin \theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

Integration by parts:

$$\int_a^b u(t)v(t) dt = u(t)v(t)\Big|_a^b - \int_a^b v(t)u'(t) dt$$

Fourier Series:

$$x(t) = \sum_{k=-\infty}^{\infty} \alpha_k e^{j2\pi\left(\frac{k}{T_0}\right)t}, \quad \alpha_k = \frac{1}{T_0} \int_0^{T_0} x(t) e^{-j2\pi\left(\frac{k}{T_0}\right)t} dt$$

Convolution:

$$h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[n-k]x[k] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

DFT:

$$x[n] = \sum_{k=0}^{N-1} X[k] e^{j\left(\frac{2\pi k}{N}\right)n}, \quad X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j\left(\frac{2\pi k}{N}\right)n}, \quad k = 0, \dots, N-1$$

Z-transform:

$$X(z) = \sum_k x[k] z^{-k}$$