Solutions to EECS 206 Exam 1, 2006-2-10

(There were multiple versions of the exam so the solutions below may not be in the same order as your exam.)

1. (a). $(1 + \jmath\sqrt{3}) + (-1 - \jmath\sqrt{3}) - 1 = -1$.

2. (d). $\mathrm{Re}\{(\sqrt{3} - \jmath)(1 + \jmath\sqrt{3})\} = 2\sqrt{3}$

3. (a). $2 = \mathrm{Im}\{(x + \jmath\sqrt{3})\,\mathrm{e}^{-\jmath\pi/2}\} = \mathrm{Im}\{(-\jmath x + \sqrt{3})\} = -x$

4. (a). $\left(2\,\mathrm{e}^{-\jmath\pi/6}\right)^{11} = 2^{11}\,\mathrm{e}^{-\jmath 11\pi/6} = 2^{11}\,\mathrm{e}^{\jmath\pi/6}$.

5. (c). $3\cos(5t + \pi/2) + \cos(5t - \pi/2) = 2\cos(5t + \pi/2)$.

6. (b). $\cos(\pi t - \pi/2) + \cos(\pi t + \pi/6) \Rightarrow -\jmath + \mathrm{e}^{\jmath\pi/6} = \mathrm{e}^{-\jmath\pi/6}$.

7. (a). $1 + A\,\mathrm{e}^{\jmath\pi/7} = M\,\mathrm{e}^{\jmath\phi}$ for some $0 \le \phi < \pi/7$.

8. (e). $\mathrm{MS}(x + y) = 6 \cdot (1/2)^2 = 3/2$.

9. (e). $\mathrm{M}(x) = \frac{1}{2}\int_0^2 x_9(t)\,\mathrm{d}t = \frac{1}{2}\,[1 + 1 + 1/2] = 5/4$.

10. (d). Time scaling $t/2$ slows the signal down by a factor of two, so the period is $2 \cdot 2 = 4$.

11. (c). The signal is mostly near 1 and sometimes near 2, so the mean squared value is between 1 and 4. Precisely: $\mathrm{MS}(x) = \frac{1}{2}\int_0^2 x_9^2(t)\,\mathrm{d}t = \frac{1}{2}\left(1 + \int_1^2 t^2\,\mathrm{d}t\right) = \frac{1}{2}\left(1 + \frac{1}{3}(2^3 - 1^3)\right) = 5/3$.

12. (a). $1 \le 1 + t/2 \le 3 \Rightarrow 0 \le t/2 \le 2 \Rightarrow 0 \le t \le 4$

13. (e). $\mathrm{E}(y) = \mathrm{E}(x) = 1 + \frac{1}{2}2^2 = 3$.

14. (b). $\mathrm{MS}(x_{12}) = \frac{1}{2}\left[1 + \frac{1}{2}2^2\right] = 3/2$.

15. (d). $T_0 = \mathrm{LCM}(1/4, 1/6) = 1/2 \Rightarrow f_0 = 2$ kHz.

16. (b). $\mathrm{RMS}(x_{15}) = \sqrt{4^2 + 2 \cdot (2\sqrt{2})^2 + 2 \cdot 1^2} = \sqrt{34} \approx 5.8$.

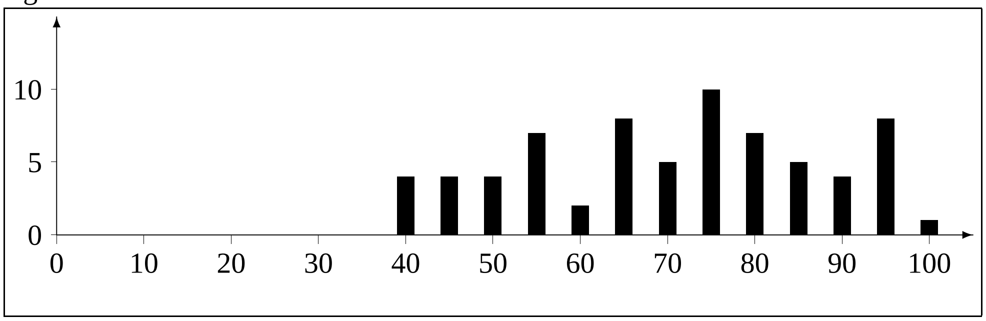
17. (e). The output signal is $4 + 4\sqrt{2}\cos(2\pi 4000t - 3\pi/4)$.

18. (b). The difference signal is $x_{15}(t) - y(t) = 2\cos(2\pi 6000t + \pi/3)$ and its RMS value is $\sqrt{2}$.

19. (e). $x(t) = 4 + 4\sqrt{2}\cos(2\pi 4000t - 3\pi/4) + 2\cos(2\pi 6000t + \pi/3)$

20. (d). $8\cos^3(y) = (\mathrm{e}^{\jmath y} + \mathrm{e}^{-\jmath y})^3 = \sum_{k=0}^{3}\begin{pmatrix}3\\k\end{pmatrix}\mathrm{e}^{\jmath ky}\,\mathrm{e}^{-\jmath(3-k)y} = \sum_{k=0}^{3}\begin{pmatrix}3\\k\end{pmatrix}\mathrm{e}^{\jmath(2k-3)y}$, by the binomial theorem, where $2k - 3 = \{-3, -1, 1, 3\}$. (Or just expand it out without binomial theorem.)

Section 001: 69 students, mean=70.6, median=75, std=16.8, 1 student scored 100%, 8 students scored below 50%.
Section 002: 73 students, mean=63.2.
EECS 398 (AOSS): 18 students, mean=48.3.
Section 001 histogram:



For elaboration on these solutions, please come to office hours.