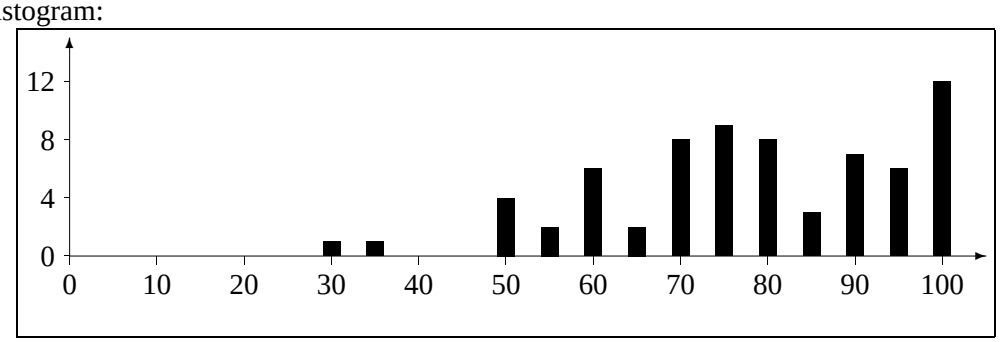
Solutions to EECS 206 Exam 2, 2006-3-17

1. (c). nonlinear and time-invariant
2. (b). linear and *not* time-invariant
3. (c). not stable and causal
4. (a). stable and causal
5. (e). $0.7\pi = 2\pi(7/20)$
6. (d). $0.7\pi = 2\pi(7/20)$ and $\frac{2}{15}\pi = 2\pi(1/15)$ and LCM(20,15) = 60.
7. (c). $270\pi/50 = 27\pi/5 \equiv -3\pi/5 = -30\pi/50$
8. (c). $x[n] = x(n/50)\,.\,\sin(50\pi n/50) = 0$ and $\cos(270\pi n/50) = \cos(30\pi n/50) \rightarrow \boxed{\text{ideal}} \rightarrow \cos(30\pi t)\,.$

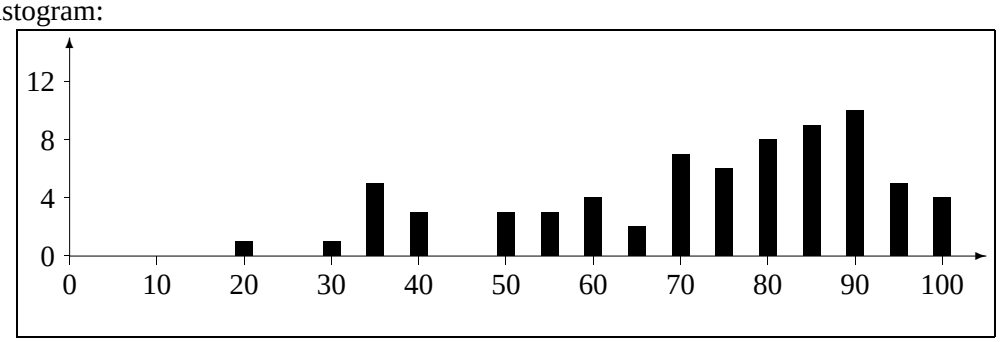
9. (f). The highest frequency component is 100+50 = 150 Hz, so we must sample at a rate higher than twice that, or $f_s > 300$ Hz.
10. (f). in the absence of aliasing: $4\sqrt{2}\cos(2\pi 10t - 3\pi/4) + 2\cos(30\pi t + \pi/3)$ or in the presence of aliasing, possibly: $4\sqrt{2}\cos(2\pi 50t - 3\pi/4) + 2\cos(30\pi t + \pi/3)$
11. (c). Only the components at $\pm 3\pi/4$ remain, and $1^2 + 1^2 = 2$.
12. (b). $2\cos\left(\frac{4}{3}\pi n\right) = 2\cos\left(-\frac{2}{3}\pi n\right) = 2\cos\left(\frac{2}{3}\pi n\right) = \mathrm{e}^{\jmath\frac{2}{3}\pi n} + \mathrm{e}^{-\jmath\frac{2}{3}\pi n}\,.$
13. (d). $X[k] = \frac{1}{4}\left(4 + 8\,\mathrm{e}^{-\jmath\frac{2\pi}{4}k} + 8\,\mathrm{e}^{-\jmath\frac{2\pi}{4}2k} + 8\,\mathrm{e}^{-\jmath\frac{2\pi}{4}3k}\right) = 1 + 2(-1)^k + 4\cos\left(\frac{\pi}{2}k\right) = \{\underline{7}, -1, -1, -1\}\,.$
14. (e). The 3-point DFT of $x[n]$ is $X[k] = \frac{1}{3}\left(3 + 6\,\mathrm{e}^{-\jmath\frac{2\pi}{3}k} + 6\,\mathrm{e}^{-\jmath\frac{3\pi}{3}2k}\right) = 1 + 4\cos\left(\frac{2\pi}{3}k\right) = \{\underline{5}, -1, -1\}\,.$
So $x[n] = 5 - \mathrm{e}^{\jmath\frac{2\pi}{3}n} - \mathrm{e}^{\jmath\frac{2\pi}{3}2n} = 5 - 2\cos\left(\frac{2\pi}{3}n\right)\,.$
Alternatively, plug $n = 0$ into the various answers; only (e) gives the correct value $x[0] = 3$.

15. (f). $D = X[3] = \frac{1}{4}\left[30 + 20\,\mathrm{e}^{-\jmath\frac{\pi}{2}3} + 10\,\mathrm{e}^{-\jmath\pi 3} + 0\right] = \frac{1}{4}\left[30 + 20\jmath - 10\right] = 5 + 5\jmath$
16. (f). $\sum_k |X[k]|^2 = 9$
17. (a). $x[n] = \delta[n] + 2\,\delta[n-2] \;\Rightarrow\; y[n] = x[n] + x[n-2] = \delta[n] + 2\,\delta[n-2] + \delta[n-2] + \delta[n-4]\,.$
18. (b). Cascades of LTI systems are LTI; cascades of FIR systems are FIR, cascades of causal systems are causal, and all FIR systems are stable.
19. (a). $(\delta[n] + \delta[n-2]) * (\delta[n] + 2\,\delta[n-1]) = \delta[n] + 2\,\delta[n-1] + \delta[n-2] + 2\,\delta[n-4]\,.$
20. (d). $y[n] = x[n] + x[n-2] = \cos(\pi n) + \cos(\pi(n-2)) = \cos(\pi n) + \cos(\pi n) = 2\cos(\pi n)\,.$

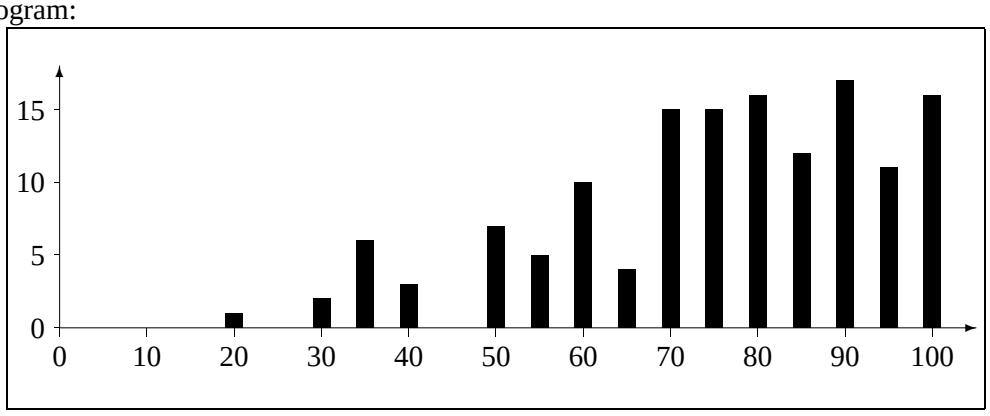
Section 001: 69 students, mean=78.2, median=80, std=17.1
Section 002: 71 students, mean=72.5, median=80, std=20.2
EECS 398 (AOSS): 18 students, mean=60.0, median=57.5 , std=18.6
Combined 206: 140 students, mean=75.3, median=80, std=18.9
Section 001 histogram:



Section 002 histogram:



Combined histogram:



For elaboration on these solutions, please come to office hours.