Solutions to EECS 206 Exam 3, 2006-4-20

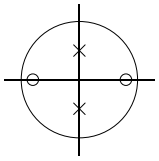
1. (c) $\frac{1}{1+z^{-1}} + \frac{1}{1+2z^{-1}} = \frac{2+3z^{-1}}{1+3z^{-1}+2z^{-2}} = \frac{2z^2+3z}{z^2+3z+2}$
2. (d) $1 + 2z^{-1} + \frac{1}{1-z^{-1}} = \frac{2+z^{-1}-2z^{-2}}{1+z^{-1}}$
3. (d) $\{\underline{1}, 2, 2, 2, \ldots\} = \delta[n] + 2\,u[n-1] \Longleftrightarrow 1 + 2\frac{z^{-1}}{1-z^{-1}} = \frac{1+z^{-1}}{1-z^{-1}}$.
4. (c) $\frac{2z-3}{z(z-1)} = \left(2 - 3z^{-1}\right)\frac{z^{-1}}{1-z^{-1}} \Rightarrow \{\underline{2}, -3\} * u[n-1]$. Also $\frac{2z-3}{z(z-1)} = \frac{3}{z} + \frac{-1}{z-1} \Rightarrow 3\,\delta[n-1] - u[n-1]$.
5. (a) $H(z) = (1 - 5z^{-1} + 6z^{-2})/(1 - 3z^{-1}) = 1 - 2z^{-1} \Rightarrow h[n] = \{\underline{1}, -2\}$.
6. (d) $H(z) = (1 + z^{-1} + z^{-2})/(1 + z^{-1} - 6z^{-2})$, which has poles at $z = -3$ and $z = 2$.

7. (b) $H(z) = \frac{1+z^{-2}}{1-z^{-2}} \Rightarrow \mathcal{H}(\pi/4) = \frac{1+\mathrm{e}^{-\jmath\,\pi/2}}{1-\mathrm{e}^{-\jmath\,\pi/2}} = \frac{1-\jmath}{1+\jmath} = \mathrm{e}^{-\jmath\,\pi/2}$.
8. (b) $H(z) = \frac{1+z^{-2}}{1-z^{-2}} \Rightarrow \mathcal{H}(\pi/2) = \frac{1+\mathrm{e}^{-\jmath\,\pi}}{1-\mathrm{e}^{-\jmath\,\pi}} = 0$. (Zeros at $\mathrm{e}^{\pm\jmath\,\pi/2}$.)
9. (a) $H(z) = \frac{1+z^{-2}}{1-z^{-2}} \Rightarrow \mathcal{H}(3\pi/4) = \frac{1+\mathrm{e}^{-\jmath\,3\pi/2}}{1-\mathrm{e}^{-\jmath\,3\pi/2}} = \frac{1+\jmath}{1-\jmath} = \mathrm{e}^{\jmath\,\pi/2}$.
10. (b) causal, linear, IIR. but poles at ± 1, not outside unit circle, so not BIBO stable.
11. (a) $H(z) = \frac{1+z^{-2}}{1-z^{-2}} \Rightarrow \mathcal{H}(\hat{\omega}) = \frac{1+\mathrm{e}^{-\jmath\,2\omega}}{1-\mathrm{e}^{-\jmath\,2\omega}} = \frac{\mathrm{e}^{\jmath\,\omega}+\mathrm{e}^{-\jmath\,\omega}}{\mathrm{e}^{-\jmath\,\omega}-\mathrm{e}^{\jmath\,\omega}} = \frac{2\cos\omega}{2\jmath\sin\omega} = -\jmath\,\cot\omega$.
12. (c) $H(z) = \frac{1+z^{-2}}{1-z^{-2}} \Rightarrow \mathcal{H}(\hat{\omega}) = \frac{1+\mathrm{e}^{-\jmath\,2\omega}}{1-\mathrm{e}^{-\jmath\,2\omega}} \Rightarrow \mathcal{H}(\pi/4) = \frac{1-\jmath}{1+\jmath} = \mathrm{e}^{-\jmath\,\pi/2}$
$\Rightarrow y[n] = \cos\left(\frac{\pi}{4}n - \frac{\pi}{2}\right) \Rightarrow \mathrm{MS}(y) = 1/2$.

13. (e) $H(z) = \frac{4z^{-1}}{1-z^{-2}} = 4\frac{z^{-1}}{(1-z^{-1})(1+z^{-1})} = 4\frac{z}{z^2-1}$.
14. (f) $H(z) = 4\frac{z^{-1}}{1-z^{-2}} = 4\frac{z^{-1}}{(1-z^{-1})(1+z^{-1})} = \frac{2}{1-z^{-1}} + \frac{-2}{1+z^{-1}} \Rightarrow h[n] = 2\,u[n] - 2(-1)^n\,u[n]$.
15. (a) $H(z) = \frac{z}{(z-\jmath)(z+\jmath)} = \frac{z}{z^2+1}$.
16. (d) $H(z) = \frac{z}{(z-\jmath)(z+\jmath)} = \frac{z}{z^2+1} = \frac{z^{-1}}{1+z^{-2}} \Rightarrow y[n] = -\,y[n-2] + x[n-1]$.
17. (d) $Y(z) = H(z)\,X(z) = \frac{z}{z+1}\frac{1}{1-z^{-1}} = \frac{1/2}{1+z^{-1}} + \frac{1/2}{1-z^{-1}} \Rightarrow y[n] = \frac{1}{2}(-1)^n\,u[n] + \frac{1}{2}\,u[n] \Rightarrow y[2] = 1$
18. (d) $H(z) = H_1(z)\,H_2(z) = \left(\frac{1}{z} + \frac{z}{z-1}\right)\frac{z-1/2}{z+1}$, so (three) poles at $\{0, 1, -1\}$.

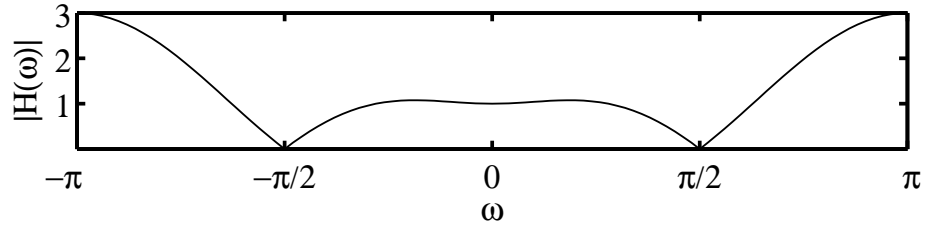
19. (b) $H(z) = z/(z+1) \Rightarrow h[n] = (-1)^n\,u[n] \Rightarrow h[3]\,/\,h[0] = -1$
20. (f) It will eliminate any signal with period $N = 4$.
21. (f) In general, use: $g\,\{\underline{1}, -2\cos(\omega_0), 1\}$. Here, $\omega_0 = 2\pi 5/30 = \pi/3$ so use $\{\underline{g}, -g, g\}$.
22. (c) $H(z) = \frac{1}{1-z^{-1}} + \frac{1}{1-2z^{-1}} = \frac{2-3z^{-1}}{1-3z^{-1}+2z^{-2}} \Rightarrow y[n] - 3\,y[n-1] + 2\,y[n-2] = 2\,x[n] - 3\,x[n-1]$.
23. (e) $\mathcal{H}(\hat{\omega}) = \frac{3}{\mathrm{e}^{\jmath\,2\omega}-5\mathrm{e}^{\jmath\,\omega}+1} \Rightarrow H(z) = \frac{3z^{-2}}{1-5z^{-1}+z^{-2}} \Rightarrow y[n] = 5\,y[n-1] - y[n-2] + 3\,x[n-2]$.
24. (d) $Y(z) = \sum_n y[n]\,z^{-n} = \sum_n \left((-1)^n\,x[n]\right)z^{-n} = \sum_n x[n](-z)^{-n} = X(-z)$

25. $H(z) = \frac{1-0.64z^{-2}}{1+\frac{1}{4}z^{-2}} = \frac{(z-0.8)(z+0.8)}{(z-\frac{1}{2}\jmath)(z+\frac{1}{2}\jmath)}$. Solution must be exact: no partial credit.

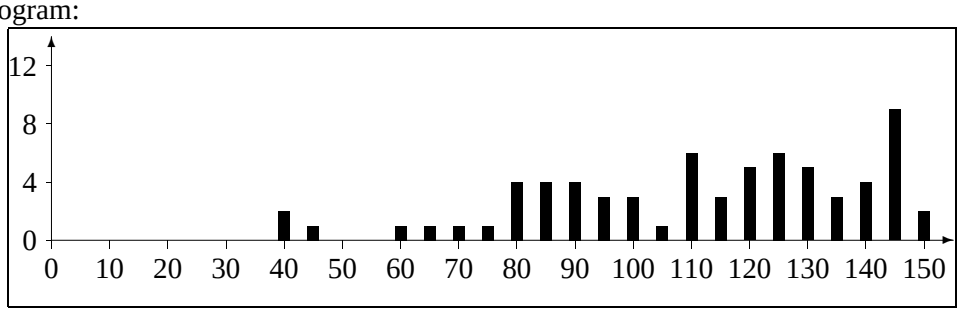


26. (c) bandpass

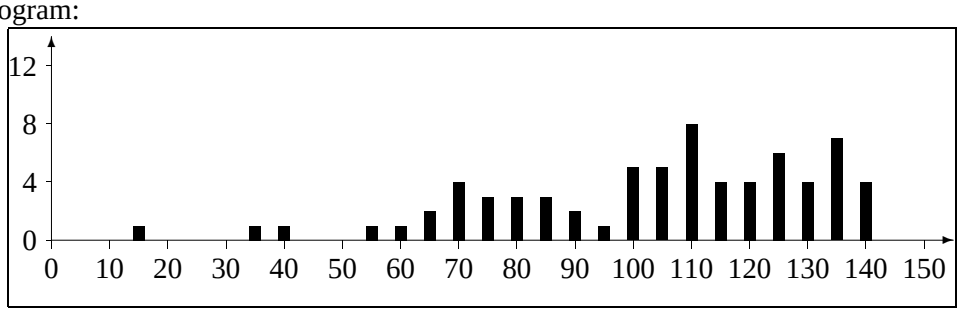
27. (e) $X[k] = \frac{1}{2}(6) = \{\underline{3}, 3\} \Rightarrow x[n] = 3 + 3\cos(\pi n)$. $\mathcal{H}(\hat{\omega}) = \frac{1}{1-\frac{1}{2}z^{-1}} \Rightarrow \mathcal{H}(0) = 2,\ \mathcal{H}(\pi) = 2/3.$
Thus $y[n] = 6 + 2\cos(\pi n)$.
28. $H(z) = \frac{(z+j)(z-j)(z-0.5)}{z^3} = (1+z^{-2})(1-0.5z^{-1}) = 1 - 0.5z^{-1} + z^{-2} - 0.5z^{-3}$
The dip at $\omega = 0$ is very subtle, so we did not grade for that. We looked for nulls at $\pm\pi/2$ and that the DC response is about 3 times smaller than the response at $\pm\pi$.



Section 001: 69 students, mean=111.2, median=115, std=28.0
Section 002: 70 students, mean=103.3, median=109, std=27.9
Combined 206: 139 students, mean=107.2, median=110, std=28.1
Section 001 histogram:



Section 002 histogram:



Combined histogram: