EECS 206

Homework Problems:

Problems from Chapter 3

Problems from the Chapter 3 section of the CD ROM

Additional questions about spectra and Fourier series

1. (a) Find the spectrum of the signal

$$x(t) = A\cos(2t)\sin(3t)$$

   (b) Find its bandwidth.

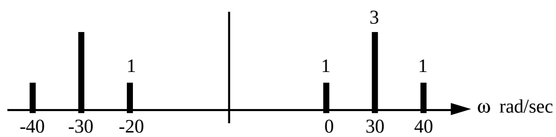
2. Find the T_o-second Fourier coefficients of the following signals, where T_o is the fundamental period of the signal.

   (a) $x(t) = t, \; 0 \le t \le T_o$

   (b) $x(t) = |\sin(t)|$

   (c) $x(t) = \sin^2(t)$

3. Consider the signal $x(t)$ whose spectrum is shown below. (All terms are real.)



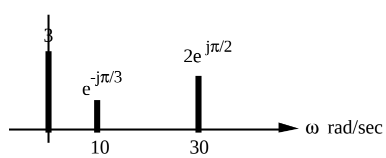
   (a) Is the signal periodic? If so, find its fundamental period.

   (b) Find its DC value.

   (c) Find its power.

   (d) Find $x(t)$. (Express the answer as a sum of sinusoids in standard form.)

4. The nonnegative frequency portion of the spectrum of a signal $x(t)$ is shown below



   (a) Is the signal periodic? If so, find its fundamental period.

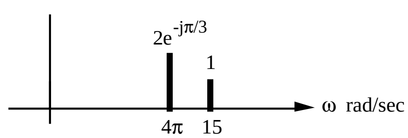
   (b) Find its DC value.

   (c) Find its power.

   (d) Find the negative frequency portion of the spectrum of $x(t)$.

   (e) Find $x(t)$. (Express the answer as a sum of sinusoids in standard form.)

5. The nonnegative frequency portion of the spectrum of a signal $x(t)$ is shown below



   (a) Is the signal periodic? If so, find its fundamental period.

(b) Find its DC value.

(c) Find its power.

(d) Find the negative frequency portion of the spectrum of $x(t)$.

(e) Find $x(t)$. (Express the answer as a sum of sinusoids in standard form.)

6. Find the power of the signal

$$x(t) = 4 + 3\cos(3t+.1) + 5\cos(7t+.3) - 4\cos(9t+.2)$$

7. Do the signals $x(t)$ and $y(t)$ given below have overlapping spectra? That is, is there a spectral component of one whose frequency lies on top of one spectral component or between the frequencies of two spectral components of the other?

$$x(t) = \cos(20t) + \cos(22t) + \cos(23t)$$

$$y(t) = 2 + \cos(10t)\cos(11t)$$

8. Find an expression for the bandwidth of the square wave that is periodic with period T such that

$$x(t) = \begin{cases} 1, & -T_1/2 \le t \le T_1/2 \\ 0, & \text{else} \end{cases}$$

where $T_1 < T$.

9. Let $x(t)$ be a periodic signal with fundamental period T_o, and let α_k be the T_o-second Fourier coefficients of $x(t)$. Suppose we also calculate the $2T_o$-second Fourier coefficients, denoted α_k^2. Derive an expression for α_k^2 in terms of α_k.

10. (a) Show that if $x(t)$ is an even periodic function, i.e. if $x(-t) = x(t)$, then all of its Fourier coefficients are real.

(a) Show that if $x(t)$ is an odd periodic function, i.e. if $x(-t) = -x(t)$, then all of its Fourier coefficients are imaginary.

11. Let $x(t)$ and $y(t)$ be sinusoids with different frequencies and support $(-\infty,\infty)$. Show that the power of $x(t)+y(t)$ equals the sum of the powers of $x(t)$ and $y(t)$. (Do not assume that $x(t)$ and $y(t)$ are such that $x(t)+y(t)$ is periodic. This fact implies that Parseval's theorem can be used to compute the power of a sum of sinusoids, even if the sum is not periodic.)

12. Derive the linearity property of Fourier series: Suppose $x(t)$ and $y(t)$ are periodic with period T and with α_k and β_k as their T-second Fourier coefficients, respectively. Then the T-second Fourier coefficients of $x(t) + y(t)$ are $\alpha_k+\beta_k$.

13. Derive the time-shifting property of Fourier series: If $x(t)$ has Fourier coefficients α_k, then $x'(t) = x(t-t_o)$ has Fourier coefficients

$$\alpha_k' = \alpha_k \, e^{-j\frac{2\pi}{T}kt_o}$$

14. Derive the frequency-shifting property of Fourier series: If $x(t)$ has Fourier coefficients α_k, then $x'(t) = x(t)\, e^{j\frac{2\pi}{T}k_o t}$ has Fourier coefficients

$$\alpha_k' = \alpha_{k-k_o} .$$

15. Derive the time-scaling property of Fourier series: Let $a>0$. If $x(t)$ is periodic with period T with T-second Fourier coefficients α_k, then $x'(t) = x(at)$ is periodic with period T/a and T/a-second Fourier coefficients

$$\alpha_k' = \alpha_k .$$

16. Let $x(t)$ be signal with support $[0,T]$, let α_k be the T-second Fourier coefficients for $x(t)$, and let α_k' denote the $2T$-second Fourier coefficients of $x(t)$. Show that

$$\alpha_{2k}' = \frac{1}{2}\alpha_k .$$