EECS 401: Ninth Problem Assignment

Due by 5PM, Fri., Mar. 30, 2007 in Changhun's mailbox in Room 2420 EECS.

Problem 1: (21 points) The hitherto uncaught burglar is hiding in city A (with a priori probability 0.3) or in city B (with a priori probability 0.6), or he has left the country. If he is in city A and N_A men are assigned to look for him there, he will be caught with probability $1 - f^{N_A}$. If he is in city B and N_B men are assigned to look for him there, he will be caught with probability $1 - f^{N_B}$. If he has left the country, he won't be captured.

Policemen's lives being as hectic as they are, N_A and N_B are independent random variables described by the probability mass functions

$$p_{N_A}(n) = \frac{2^N e^{-2}}{N!} \quad N = 0, 1, 2, \ldots$$
$$p_{N_B}(n) = \left(\frac{1}{2}\right)^N \quad N = 1, 2, 3, \ldots$$

1. What is the probability that a total of three men will be assigned to search for the burglar?
2. What is the probability that the burglar will be caught? (All series are to be summed.)
3. Given that he was captured in a city in which exactly K men had been assigned to look for him, what is the probability that he was found in city A?

Problem 2: (12 points) A number p is drawn from interval $[0, 1]$ according to the uniform distribution, and then a sequence of independent Bernoulli trials is performed, each with success probability p. What is the mean and the variance of the number of successes in k trials?

Problem 3: (12 points) We are given two independent Bernoulli processes with parameters p_1 and p_2. A new process is defined to have a success on the kth trial ($k = 1, 2, 3, , \ldots$) only if *exactly* one of the other two processes has a success on its kth trial.

1. Determine the PMF for the number of trials up to and including the rth success in the new process.
2. Is the new process a Bernoulli process?

Problem 4: (15 points) Determine the expected value, variance and moment generating function for the total number of trials from the start of a Bernoulli process up to and including the nth success after the mth failure.

Problem 5: (10 points) Based on your understanding of the Poisson process, determine the numerical values of a and b in the following expression and explain your reasoning:

$$\int_t^\infty \frac{\lambda^6 \tau^5 e^{-\lambda\tau}}{5!}\, d\tau \;=\; \sum_a^b \frac{(\lambda t)^k e^{-\lambda t}}{k!}.$$

Problem 6: (30 points) You are visiting the rainforest, but unfortunately your insect repellent has run out.

1. As a result, at each second, a mosquito lands on your neck with probability 0.2.
   (a) What's the PMF for the time until the first mosquito lands on you?
   (b) What's the expected time until the first mosquito lands on you?
   (c) What if you weren't bitten for the first ten seconds – what would be the expected time until the first mosquito lands on you (from time t=10)?
2. Instead, imagine the rainforest had only one mosquito, which arrived in the following way: the time of arrival is exponentially distributed with $\lambda = 0.2$.
   (a) What's the expected time until the first mosquito lands on you?
   (b) What if you weren't bitten for the first ten seconds – what would be the expected time until the first mosquito lands on you (from time t=10)?