FALL 2012 EECS 598-03: Fast Multipole Methods and Integral Equation Solvers

Instructor: Eric Michielssen (EECS)

Time & Location: 1:30-3:00 Mondays and Wednesdays EECS 3427

Overview: This course focuses on integral equation methods for solving the classical partial differential equations of mathematical physics along with fast algorithms that effect their iterative solution. Our focus will be fast (multipole) algorithms for low-rank-under-compression kernels (e.g. Poisson), and Green functions for the Helmholtz and time-dependent wave equations (including their Maxwell equation extensions). Additional topics include FFT and Butterfly-accelerated solvers, and recent developments in direct solvers. At the end of the course, students should be able to comfortably read current fast algorithm literature and use fast multipole constructs in their own research.

Grading: based on class participation, mid semester, and final projects.

Tentative syllabus:

Volume and boundary integral equations. Laplace equation; Yukawa/screened-Coulomb/modified-Helmholtz equation; Time-dependent wave equation and scalar-Helmholtz/time-harmonic-wave equation; Time-dependent Maxwell equations and vector-Helmholtz/time-harmonic-Maxwell equations; Applications in classical mechanics: electro- and magnetostatics, acoustics, electrodynamics, many-body dynamics.

Iterative solution techniques. Simplest/stationary: Neumann/Born series and Chebyshev approximations; Krylov-subspace-based/non-stationary: Generalized Minimum RESidual (GMRES) and Conjugate Gradient (CG) methods; Singular Value Decompositions (SVDs); Interpolation.

Fast methods for applying low-rank-under-separation linear operators. Some history: from Barnes-Hut to the early fast-multipole methods (FMMs); Classical FMMs for the Poisson equation; Numerical representations based on algebra and on exponentials/plane-waves. Adaptive Cross Approximation. Application to capacitance and induction extraction.

Fast multipole method (FMM) for the Helmholtz and time-Harmonic Maxwell equations**.** Bessel functions and partial wave expansions; Low frequency, high frequency, and wideband implementations; Directional/windowed translation operators. Butterfly operator compression. Application to large scale acoustic and electromagnetic scattering and radiation phenomena.

Fast Plane-Wave Time-Domain (PWTD) algorithm for the solution of time-dependent wave equations. Whitaker expansions. Low frequency, high frequency, and wideband implementations. Application to various time-dependent electromagnetic scattering and guidance phenomena, and electromagnetic compatibility.

Direct solution techniqu*es*

Course materials: Instructor notes, historically important and current literature, and several fast algorithm/multipole texts available on the web.

- W. C. Chew, J.-M. Jin, E. Michielssen, and J. Song, "Fast and Efficient Algorithms in Computational Electromagnetics," Artech House, Norwood, MA.
- Per-Gunnar Martinsson and Mark Tygert. Multilevel Compression of Linear Operators: Descendants of Fast Multipole Methods and Calderon-Zygmund Theory – available on Mark Tygert's website: http://pantheon.yale.edu/~mwt7/gradcourse/notes.pdf. This course will in part be modeled after the one he teaches at Yale.
- Rick Beatson and Leslie Greengard, A short course on fast multipole methods, available on Leslie Greengard's website: http://math.nyu.edu/faculty/greengar/shortcourse_fmm.pdf.