Final overview

EECS 477

Lecture 24, 12/10/2002

Asymptotic notation

- $O(n)$, $\Omega(n)$, $\Theta(n)$
  – Know them by heart
- Average, best, and worst performance
- Elementary operations
- Lower bounds and upper bounds
- Problems, algorithms, instances
- Barometers, simple control structures

Recurrencies

- Recursion, divide and conquer
- Inhomogeneous recurrence
  - General form of solution
  - Taking care of the RHS and initial conditions
  - Multiple roots of characteristic equation
  - Transformations
    - Change of variable
    - Change of range

Master Theorem

- $T(n) = a\,T(n/b) + f(n)$
- Three cases
  - ❖ $f(n) = O(n^{\log_b a-\varepsilon})$ then $T(n) = \Theta(n^{\log_b a})$
  - ❖ $f(n) = \Theta(n^{\log_b a-\varepsilon})$ then $T(n) = \Theta(n^{\log_b a} \log n)$
  - ❖ $f(n) = \Omega(n^{\log_b a+\varepsilon})$ (and if $af(n/b) \leq cf(n)$ for some constant $c<1$) then $T(n) = \Theta(f(n))$
- If <= then can claim O result
- There are gaps in the theorem

Data structures

- Heaps (various)
- Disjoint sets
- Trees
- Hash tables
- Vectors and other simple ones

Graphs

- Complete, connected, strongly connected, articulation points, spanning trees, paths, cycles, DAGs
- Two simple traversals
  - DFS, BFS: What is runtime asymptotics?
- Two representations
- Two parameters

Graphs with edge weights

- MST: O(E log V)
  - Kruskal: shortest edges first
  - Prim: grow a tree
- Dijkstra: O((A+N) log N) or $O(N^2)$
  - Shortest paths from a vertex to everybody else
- Asymptotics
  - Two parameters!

Knapsack: variants

- Greedy
  - Optimal for breakable objects
- Dynamic programming
  - Types of objects
- NP-complete
  - Non-breakable objects
- Approximate algorithm
  - Modifies the greedy

Greedy algorithms

- Scheduling
  - Min time in the system
  - Unit time jobs with with deadlines
- Making change
  - Optimal for some coinage
- Often
  - Basis for heuristic algorithms
    - Coloring, metric TSP

Divide and conquer

- Long integer arithmetic

  a*b MT, different lengths
- Strassen
  - Matrix multiplication
- Median in linear time
- Mergesort

Dynamic programming

- Shortest paths from everybody to everybody
  - Floyd's algorithm
- Chained matrix multiplication
- Table
  - Making change
  - Knapsack
- Memory functions

Traveling salesman: variants

- Trivial n! permutations
- Dynamic programming: $\Theta(n^2 2^n)$
  - Builds Opt[S; x] = length of the cheapest path starting in city 1 visiting all the cities in S\{x} and stopping in city x
- Branch and bound
  - Exclude/include edges one by one
    - That gives constraints
- Metric TSP

Backtracking, Branch&Bound

- Games
- Eight queens
- Assignment
  - Jobs to workers
- TSP
- Minimax

Complexity

- Lower bounds
  - Information theoretical
    - Game of K questions
    - Average leaves depth in a tree
      - Sorting complexity
  - Adversary arguments
    - Finding maximum
    - Finding median
    - Graph connectivity

P and NP

- Optimization and Decision problems
  – Classes P and NP
- Polynomial reduction
- NP-completeness
  – SAT-CNF, SAT-3-CNF
  – TSPD, HAMD, HAM
- NP-hardness

Approximate algorithms

- Knapsack
  – Modified from greedy
- Metric TSP
  – MST

Book sections

- 3, 4, 5
- 6.1-6.5, 6.6.1
- 7.1-7.6
- 8, 9
- 12.2, 12.3, 12.5
- 13.1-13.2